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# CYLINDRICAL AND SPHERICAL ION ACOUSTIC SOLITARY WAVES IN SUPERTHERMAL PLASMA WITH TWO TEMPERATURE ELECTRONS

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*Abstract*. Nonlinear ion acoustic (IA) solitary waves are investigated in a nonplanar superthermal plasma with two-temperature electrons. We derived a cylindrical/ spherical Korteweg–de Vries (KdV) and a modified KdV equation for ion acoustic waves by using the reductive perturbation technique. The influence of geometry and suprathermality effects on IA waves is investigated. As we expect for double-Boltzmann distributed electrons, solitons of both polarities can exist in this model. There exists a critical value of parameters beyond which the IA soliton collapses. Furthermore, it is evident that the increase rate of velocity and amplitude in spherical geometry is greater than in cylindrical geometry.

*Key words*: superthermal electrons, solitary waves, nonplanar plasma.

## 1. INTRODUCTION

The dynamics of ion-acoustic (IA) waves, as one of the basic wave processes in plasma, has been studied in the last several decades both theoretically and experimentally [1–16]. In absence of the dissipative effects and geometry distortion, a Korteweg-de Vries (KdV) equation describes the propagation of IA waves in a weakly nonlinear regime as shown by Washimi *et al.* [1]. Oikawa *et al.* [2] adopted an extended approach to study the interaction of such waves as the superposition of two single KdV-type solitons. Sagdeev [3] proposed an approach based on a mechanical model in which the solitary waves in unmagnetized plasmas are associated to mass points moving in a pseudopotential called the Sagdeev potential. The first experimental observation of IA solitary waves was made by Ikezi *et al.* [4]. These waves arise due to the balance between nonlinearity and dispersion effects in the media [11]. Chan and Seshadri [6] studied the modulational instability of IA waves showing that they are modulationally stable. Also, Sharma *et al.* [7] obtained a nonlinear Schrödinger (NLS) equation for IA waves, and they found that for finite ion temperature these waves are modulationally unstable. An increase of the ratio of ion to electron temperature leads to decrease of the range of the unstable region and shifts it toward the small wave numbers. Goswami and Buti [8] investigated the propagation properties of IA waves in a plasma with hot and cold electron species. Subsequently, the nonlinear IA wave dynamics theory has been developed in cases of non-Maxwellian components [17–20], various wave front geometries [21–28], degenerate particles [29, 30], finite ion temperatures [31, 32], and high order nonlinearity [33], two-temperature electrons [8, 18, 34], etc.

Watanabe [33] considered the effect of higher order nonlinearity on a soliton, and showed that a KdV equation can not explain the experimental results. There is a long tradition of experiments, where IA waves were excited with the Langmuir probes or grids [35] by applying a continuous wave (or a pulse). The modern techniques for detecting IA waves are based on laser-induced fluorescence (LIF) technique. Recently, ion-acoustic solitary waves were analyzed with this technique [36].

Ion acoustic waves can propagate in magnetized as well as unmagnetized plasmas. Gekelman and Stenzel [37] studied the IA spectra for magnetized plasma in laboratory. Yu *et al*. [38] extended the Sagdeev approach to study the IA waves in a magnetized plasma. Obliquely propagating ion-acoustic solitons in a magnetized plasma consisting of warm adiabatic ions and nonthermal electrons have been investigated by Cairns *et al.* [39]. Recently, the oblique propagation of large amplitude IA solitary waves in a magnetized superthermal dusty plasma has been discussed by Shahmansouri and Alinejad [40]. Further studies of the propagation of the IA solitary waves in a magnetized plasma may be found in [41, 42] and references therein.

The IA solitary waves in electron-positron-ion (e-p-i) plasmas have been investigated by a number of authors in unmagnetized [43, 44] and magnetized [45, 46] plasma. Popel *et al*. [43] studied the nonlinear dynamics of IA solitary waves in a three component unmagnetized plasma. The weak amplitude IA double layers have been investigated by Mishra *et al.* [47] in a multicomponent plasma with positrons.

Ion acoustic solitons in a two electron component plasma have been studied by various authors in different plasma conditions, both theoretically [10, 14, 18, 48–51] and experimentally [52]. Nishihara and Tajiri [53] studied IA waves in a plasma with cold and hot Maxwell-Boltzmann distributed electrons discovering two regimes of wave propagation in the system, normal and anomalous. Baboolal *et al.* [10], using the Sagdeev approach, investigated the formation and characteristics of the large amplitude ion acoustic solitons and double layers in plasmas containing hot and cool Maxwell-Boltzmann distributed electron fluids. Masood *et al.* [54] have studied nonplanar IA solitary waves in a two electron temperature plasma. Cairns *et al.* [55] investigated IA waves in a non-Maxwellian plasma with one electron component featuring the Cairns distribution. Other non-Maxwellian plasmas with only one electron component that were investigated contain $\kappa$ (kappa) distributed [56, 57] or Tsallis distributed electrons [58]. Recently, Schippers *et al.* [59] have analysed a combined hot and cool electron component model with both species being kappa distributed and found a best fit for the electron velocity distribution. Their work shows that plasmas composed of two $\kappa$ -distributed electron components are relevant to the description of the Saturnian magnetosphere. Baluku and Helberg used this model as base of a kinetic theory for electron acoustic waves in Saturn's magnetosphere [60], then approaching the topic of ion acoustic solitons in a plasma with two-temperature kappa-distributed electrons [18]. The isotropic kappa velocity distribution of particles has the form of [61]

$$F(\upsilon)=\frac{1}{[\pi(2\kappa-3)k_BT/m]^{3/2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}\left(1+\frac{m\upsilon^2}{(2\kappa-3)k_BT}\right)^{-\kappa-1}, \tag{1}$$

where $\kappa$ is the spectral index and measures the slope of the energy spectrum of superthermal particles which form the tail of the velocity distribution function. This distribution is defined for $\kappa>3/2$.

Recent investigations show that the one-dimensional geometry cannot account for all features of realistic plasma, as propagation characteristics of solitary waves in nonplanar plasmas significantly differ from those in unbounded planar ones. This was verified by Maxon and Viecelli in investigation of the small amplitude IA solitons in a plasma with spherical [22], and cylindrical [23] symmetry. Ion acoustic (IA) waves in the nonplanar plasmas [21, 22–28, 62, 63] have been extensively studied in the last few decades. Maxon studied the IA solitons in plasma with cylindrical and spherical symmetry [65], Ko and Kuehl gave solitary wave solutions [64] and, Sahu and Roychoudhury investigated the IA waves in a plasma with nonthermal electrons and warm ions [26] for the same symmetries.

In this work we consider the propagation of IA solitons in a nonplanar, collisionless, unmagnetized plasma comprising of adiabatic ion fluid and, cool and hot kappa distributed electron species. We suppose that the phase velocity of the IA waves is much smaller than the thermal velocity of the electrons. Then we study the influence of physical relevant parameters, such as the ion (electron) temperature and ion (electron) superthermality on the characteristic properties of IA solitary waves.

## 2. BASIC EQUATIONS

In a plasma containing adiabatic ions and a mixture of hot ($T_h$) and cold ($T_c$) kappa-distributed electrons as found in the Saturn's magnetosphere, we consider nonlinear spherical and cylindrical IA waves propagating in a radial direction. The charge neutrality condition is $n_{i0} - n_{c0} - n_{h0} = 0$, where $n_{i0}$, $n_{c0}$ and $n_{h0}$ are the unperturbed ion, cool electron and hot electron number densities, respectively.

The nonlinear dynamic of the nonplanar IA waves is governed by [26, 66]

$$\frac{\partial n_i}{\partial t} + \frac{1}{r^v}\frac{\partial}{\partial r}(r^v n_i u_i) = 0, \tag{2}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial r} = -\frac{e}{m_i}\frac{\partial \varphi}{\partial r} - \frac{1}{m_i n_i}\frac{\partial P_i}{\partial r}, \tag{3}$$

$$\nabla^2 \varphi = -\frac{e}{\varepsilon_\circ}(n_i - n_c - n_h), \tag{4}$$

where $v$ indicates the geometry of the system (one-dimensional planar geometry for $v = 0$, cylindrical geometry for $v = 1$, spherical geometry for $v = 2$). The pressure of adiabatic ions is given by $P_i = n_{i0} k_B T_i (n_i / n_{i0})^3$. Integrating the distribution (1) over the velocity space, the electron densities are

$$n_c = n_{c0}\left(1 - \frac{e\varphi}{k_B T_c(\kappa_1 - 3/2)}\right)^{-\kappa_1 + 1/2}, \tag{5}$$

$$n_h = n_{h0}\left(1 - \frac{e\varphi}{k_B T_h(\kappa_2 - 3/2)}\right)^{-\kappa_2 + 1/2}. \tag{6}$$

We consider the following normalization of the quantities to obtain non-dimensional variables: $n_j \to n_j / n_{i0}$, $u_i \to u_i / C_s$, $\varphi \to e\varphi / k_B T_c$, $r \to r / \lambda_{Di}$, $t \to t / \omega_{pi}^{-1}$. Here $C_s = \sqrt{k_B T_c / m_i}$ is the ion acoustic speed, $\lambda_{Di} = \sqrt{\varepsilon_\circ k_B T_c / n_{i0} e^2}$ is the ion Debye length and $\omega_{pi} = \sqrt{n_{i0} e^2 / m_i \varepsilon_\circ}$ is the ion plasma frequency, respectively. The normalized fluid equations take the following form

$$\frac{\partial n_i}{\partial t} + \frac{1}{r^v}\frac{\partial}{\partial r}(r^v n_i u_i) = 0, \tag{7}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial r} = -\frac{\partial \varphi}{\partial r} - 3\sigma_i n_i \frac{\partial n_i}{\partial r}, \tag{8}$$

$$\frac{1}{r^v}\frac{\partial}{\partial r}\left(r^v \frac{\partial \varphi}{\partial r}\right) = n_c + n_h - n_i, \tag{9}$$

$$n_c = f\left(1-\frac{\varphi}{\kappa_1 - 3/2}\right)^{-\kappa_1+1/2}, \tag{10}$$

$$n_h = (1-f)\left(1-\frac{\varphi}{\sigma_e(\kappa_2 - 3/2)}\right)^{-\kappa_2+1/2}, \tag{11}$$

where $f = n_{c0}/n_{i0}$, $\sigma_i = T_i/T_c$ and $\sigma_e = T_h/T_c$.

## 3. DERIVATION OF CYLINDRICAL AND SPHERICAL KdV EQUATIONS

In order to investigate the IA solitary waves in the plasma model under consideration, we employ a weakly nonlinear theory of the electrostatic waves with small but finite amplitude. The dependent variables are expanded as follows:

$$n_i = 1 + \varepsilon n_i^{(1)} + \varepsilon^2 n_i^{(2)} +, \tag{12a}$$

$$u_i = \varepsilon u_i^{(1)} + \varepsilon^2 u_i^{(2)} +, \tag{12b}$$

$$\varphi = \varepsilon\varphi^{(1)} + \varepsilon^2\varphi^{(2)} +, \tag{12c}$$

where $\varepsilon$ is the smallness parameter that measures the strength of the wave amplitude. Also the densities of cold and hot electrons can be expanded to give

$$n_h = \sum_{i=0}^{\infty} b_i\varphi^i \text{ and } n_c = \sum_{i=0}^{\infty} c_i\varphi^i ,$$

where $c_0 = f$, $b_0 = 1-f$, $c_1 = f(2\kappa_1 - 1)/(2\kappa_1 - 3)$,
$b_1 = (1-f)(2\kappa_2 - 1)/\sigma_e(2\kappa_2 - 3)$, $c_2 = f(2\kappa_1 - 1)(2\kappa_1 + 1)/[2!(2\kappa_1 - 3)^2]$,
$b_2 = (1-f)(2\kappa_2 - 1)(2\kappa_2 + 1)/[2!\sigma_e^2(2\kappa_2 - 3)^2]$
$c_3 = f(2\kappa_1 - 1)(2\kappa_1 + 1)(2\kappa_1 + 3)/[3!(2\kappa_1 - 3)^3]$,
$b_3 = (1-f)(2\kappa_2 - 1)(2\kappa_2 + 1)(2\kappa_2 + 3)/[3!\sigma_e^3(2\kappa_2 - 3)^3]$
and so on. We introduce the stretched coordinates [22, 23, 67]

$$\tau = \varepsilon^{3/2}t,, \ \xi = -\varepsilon^{1/2}(r + M_0 t) \tag{13}$$

where $M_0$ is the soliton speed. Substituting the set of Eqs. (12), along with the stretched coordinates (13), into (7–11), and collecting the terms in different powers of $\varepsilon$, the lowest order of $\varepsilon$ leads to

$$u_i^{(1)} = -M_0 n_i^{(1)}, \tag{14a}$$

$$-M_0 u_i^{(1)} = \varphi^{(1)} + 3\sigma_i n_i^{(1)}, \tag{14b}$$

$$0 = c_1\varphi^{(1)} + b_1\varphi^{(1)} - n_i^{(1)}, \tag{14c}$$

Then, the following expression can be obtain for the soliton speed

$$M_0 = \sqrt{3\sigma_i + \frac{1}{c_1 + b_1}}. \tag{15}$$

Finally for the next power of $\varepsilon$, we have

$$-M_0 \frac{\partial n_i^{(2)}}{\partial \xi} + \frac{\partial n_i^{(1)}}{\partial \tau} - \frac{\nu}{M_0 \tau} u_i^{(1)} - \frac{\partial u_i^{(2)}}{\partial \xi} - \frac{\partial}{\partial \xi}(n_i^{(1)} u_i^{(1)}) = 0, \tag{16a}$$

$$-M_0 \frac{\partial u_i^{(2)}}{\partial \xi} + \frac{\partial u_i^{(1)}}{\partial \tau} - u_i^{(1)} \frac{\partial u_i^{(1)}}{\partial \xi} = \frac{\partial \varphi^{(2)}}{\partial \xi} + 3\sigma_i \frac{\partial n_i^{(2)}}{\partial \xi} + 3\sigma_i n_i^{(1)} \frac{\partial n_i^{(1)}}{\partial \xi}, \tag{16b}$$

$$\frac{\partial^2 \varphi^{(1)}}{\partial \xi^2} = c_1\varphi^{(2)} + c_2\left(\varphi^{(1)}\right)^2 + b_1\varphi^{(2)} + b_2\left(\varphi^{(1)}\right)^2 - n_i^{(2)}. \tag{16c}$$

Combining the set of Eqs. (16), the condition for annihilation of secular terms leads to the cylindrical/spherical KdV equation

$$\frac{\partial \varphi^{(1)}}{\partial \tau} + A\varphi^{(1)} \frac{\partial \varphi^{(1)}}{\partial \xi} + B\frac{\partial^3 \varphi^{(1)}}{\partial \xi^3} + \frac{\nu}{2\tau}\varphi^{(1)} = 0, \tag{17}$$

where

$$A = \frac{(M_0^2 - 3\sigma_i)^2}{2M_0}\left[-2c_2 - 2b_2 + \frac{3(M_0^2 + \sigma_i)}{(M_0^2 - 3\sigma_i)^3}\right],$$

$$B = \frac{(M_0^2 - 3\sigma_i)^2}{2M_0}.$$

Eq. (17) describes evolution of the nonlinear ion acoustic wave in nonplanar plasma. If we set $\nu = 0$ in Eq. (17), it reduces to Eq. (8) of Ref. [18], ignoring the notation and normalization differences. Also, if we set $(\kappa_1, \kappa_2) \to \infty$ into Eq. (16), it recovers the KdV equation obtained in Ref. [54].

Sahu and Roychoudhury [26] have studied the cylindrical and spherical IA waves in a nonthermal plasma. In that study they have introduced the analytical solutions of the cylindrical/spherical KdV equations, *via* suitable coordinate transformation [68]. Similarly, two exact solutions of Eq. (16) in the cylindrical geometry are obtained:

$$\text{(a)}\ \varphi^{(1)} = \frac{1}{\tau}\left[\frac{\xi}{2A} + \frac{3M}{A}\operatorname{sech}^2\left(\sqrt{\frac{M}{4B\tau}}(\xi + 2M)\right)\right],$$

$$\text{(b)}\ \varphi^{(1)} = \frac{1}{\tau}\left[\frac{\xi}{2A} + a_0 + a_2\tanh^2\left(\sqrt{\frac{1}{\tau}}\xi\right)\right],$$

where $M$ refers to the solitary wave speed, $a_0 = 8B/A$ and $a_2 = -12B/A$. It must be also noted that the above expressions are valid for $\tau > 0$. Furthermore, the solution of Eq. (16) in spherical geometry takes this form:

$$\varphi^{(1)} = \frac{1}{\tau}\left[\frac{\xi}{A\ Ln\tau} + \frac{c}{Ln\tau}\right].$$

In a Maxwellian electron-ion (e-i) plasma, the coefficient of the quadratic nonlinearity is positive and this system always admits compressive solitons. In a plasma with hot and cold electrons, solitons of both polarities can exist. This means that the coefficient of the quadratic nonlinear term can changes sign, and rarefactive solitons appear. In the critical domain where the coefficient of the nonlinear term in KdV equation vanishes a new type of expansion must be defined. This leads to the "modified" KdV (mKdV) equation, with a cubic rather than a quadratic nonlinearity. The analytical criterion for the case of a nonthermal e-i plasma in the nonplanar geometry was determined by Sahu and Roychoudhury [26]. The validity range of the mKdV equation is obviously limited to the neighborhood of the crossover point from compressive to rarefacting solitons. It admits both compressive as well as rarefactive solitons with similar characteristics. In order to derive the mKdV equation in the critical case, we introduce the following new stretched coordinates

$$\tau = \varepsilon^3 t,\ \ \xi = -\varepsilon(r + M_0 t). \tag{18}$$

The same procedure, as given above, for the lowest order in $\varepsilon$, leads to the similar results. To the next power of $\varepsilon$, we can obtain the following result from the combination of a set of equations

$$\sigma_i = \frac{c_2 + b_2}{6(c_1 + b_1)^3} - \frac{1}{4(c_1 + b_1)}. \tag{19}$$

Note that the above equation is obtained for $A = 0$, where $A$ is the coefficient of the nonlinear term in the KdV equation (17). The next higher order in $\varepsilon$, leads to the following set of equations

$$-M_0\frac{\partial n_i^{(3)}}{\partial\xi}+\frac{\partial n_i^{(1)}}{\partial\tau}-\frac{v}{M_0\tau}u_i^{(1)}-\frac{\partial u_i^{(3)}}{\partial\xi}-\frac{\partial}{\partial\xi}(n_i^{(1)}n_i^{(2)}+n_i^{(2)}n_i^{(1)})=0, \quad (20a)$$

$$-M_0\frac{\partial u_i^{(3)}}{\partial\xi}+\frac{\partial u_i^{(1)}}{\partial\tau}-\frac{\partial}{\partial\xi}(u_i^{(1)}u_i^{(2)})=\frac{\partial\varphi^{(3)}}{\partial\xi}+3\sigma_i\frac{\partial}{\partial\xi}\left(n_i^{(3)}+n_i^{(1)}n_i^{(2)}\right), \quad (20b)$$

$$\begin{aligned}\frac{\partial^2\varphi^{(1)}}{\partial\xi^2}&=c_1\varphi^{(3)}+2c_2\varphi^{(1)}\varphi^{(2)}+c_3\left(\varphi^{(1)}\right)^3+\\&+b_1\varphi^{(3)}+2b_2\varphi^{(1)}\varphi^{(2)}+b_3\left(\varphi^{(1)}\right)^3-n_i^{(3)}.\end{aligned} \quad (20c)$$

Combining the set of Eqs. (20), we get a modified KdV equation

$$\frac{\partial\varphi^{(1)}}{\partial\tau}+C\left(\varphi^{(1)}\right)^2\frac{\partial\varphi^{(1)}}{\partial\xi}+B\frac{\partial^3\varphi^{(1)}}{\partial\xi^3}+\frac{v}{2\tau}\varphi^{(1)}=0, \quad (21)$$

where

$$C=\frac{-6M_0^2+3(M_0^2-3\sigma_i)^2(b_2+c_2)(\sigma_i+3M_0^2)}{2M_0(M_0^2-3\sigma_i)^2}-\frac{3(b_3+c_3)(M_0^2-3\sigma_i)^2}{2M_0},$$

$$B=\frac{(M_0^2-3\sigma_i)^2}{2M_0}.$$

Note that the $b_i$ coefficients contain $\sigma_e$>>1 at their denominator, thus they are small. In the limit of $b_i\approx 0$, the coefficient of the nonlinear term in the KdV equation (17), remain always positive. In this case, the present plasma model supports only compressive solitons.

## 4. NUMERICAL RESULTS

In this section the previously obtained cylindrical/spherical KdV equations and their solitary solutions will be investigated numerically. We have used the stationary solution of Eq. (17), without the term $(v/2\tau)\varphi$, as the initial condition. In this case the solution of Eq. (17) takes the following form:

$$\varphi^{(1)}=\varphi_m\text{sech}^2\left(\frac{\xi-M\tau}{L}\right), \quad (22)$$

where the amplitude and thickness of solitons are $\varphi_m=3M/A$ and $L=\sqrt{4B/M}$, respectively. Numerical solution of (17) shows that for large values of $\tau$ [ $\tau=-9.0$ ], the nonplanar ion acoustic solitons are almost similar to the planar case. This is because for the large values of $\tau$, the term $(v/2\tau)\varphi$ is no longer dominant.

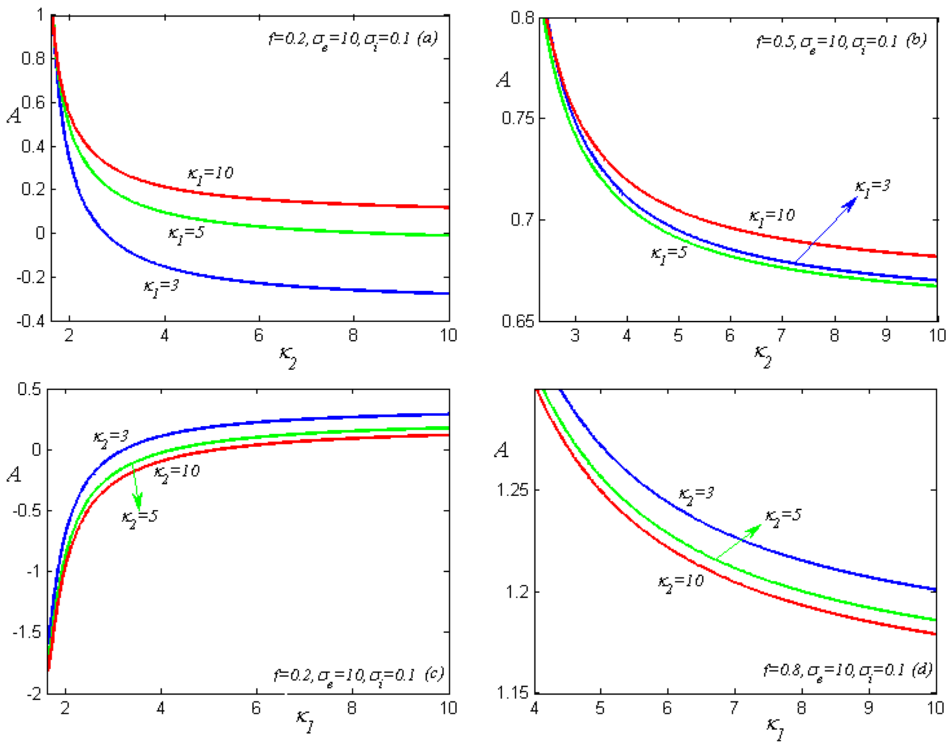


Fig. 1 – The coefficient of nonlinearity in KdV equation with respect to: a) $\kappa_2$ for $f = 0.2$; b) $\kappa_2$ for $f = 0.5$; c) $\kappa_1$ for $f = 0.2$; d) $\kappa_1$ for $f = 0.8$. Here the other parameters are $\sigma_i = 0.1$, $\sigma_e = 10$.

Figure 1 shows the variation of nonlinear term coefficient of KdV equation with respect to electrons suprathermality, for different values of the cold/hot electron population. According to Fig. 1a, when the density of the cold electrons decreases with respect to the hot electrons' density, this system can support the rarefactive solitons. When the hot electrons deviate from thermodynamic equilibrium, the coefficient of the nonlinear term changes sign and compressive solitons appear. It can be seen that the positive (negative) values of the nonlinear term coefficient increase (decrease) with $\kappa_1$, while this behavior is opposite with $\kappa_2$. Figure 1b indicates that increase of the cold electrons density leads to the compressive solitons only. The other two panels in Fig. 1 refer to variation of the nonlinear term coefficient of KdV equation with respect to the cold electrons suprathermality. These two figures also confirm that the presence of more cold electrons leads to the compressive solitons. The exact solution of the cylindrical KdV equation is plotted in Fig. 2, for different values of the hot/cold electron superthermality.

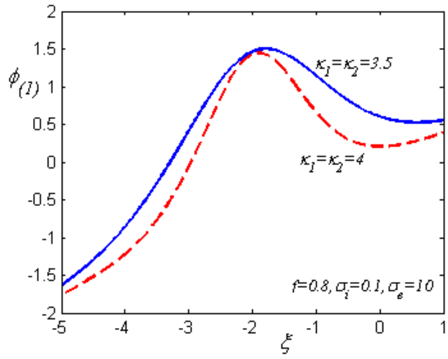


Fig. 2 – The analytical solution of the cylindrical KdV equation for $f = 0.8$, $\sigma_i = 0.1$, $\sigma_e = 10$, $\kappa_1 = \kappa_2 = 4$ (dashed line) and $\kappa_1 = \kappa_2 = 3.5$ (solid line).

Numerical solution of Eq. (17) in cylindrical and spherical coordinates, for different values of $\kappa$ (where $\kappa_1 = \kappa_2 = \kappa$) in cylindrical ($\nu = 1$) and spherical ($\nu = 2$) geometries are depicted in Fig. 3a and Fig. 3b, respectively. These figures indicate a dependency of the amplitude and width of the rarefactive IA solitons on the electron suprathermality. Both the amplitude and the width of solitons decrease when the suprathermality effects become more important *via* decrease of the kappa values. Also it is obvious that the increase rate of velocity and amplitude in spherical geometry is greater than in cylindrical geometry.

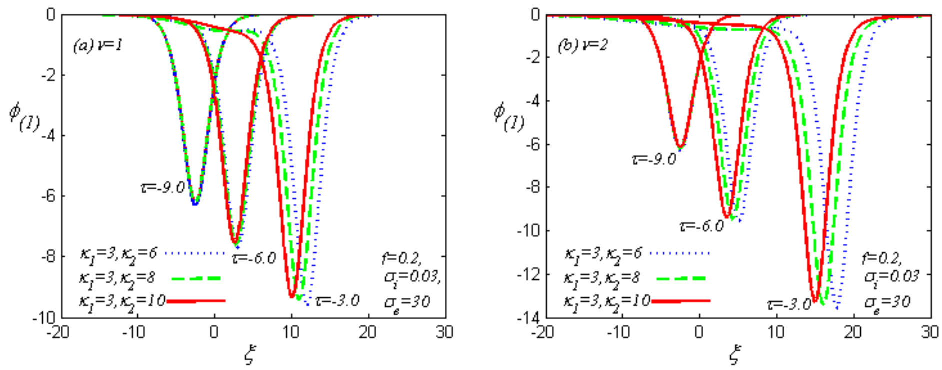


Fig. 3 – Numerical solution of Eq. (17) for several values of $\tau$ ranging from −9 to −3, and different values of $\kappa$ ; a) in cylindrical ($\nu = 1$) geometry; b) in spherical ($\nu = 2$) geometry. The other parameters are $\sigma_e = 30$, $\sigma_i = 0.03$ and $f = 0.2$.

The compressive IA solitary waves are shown in Fig. 4, for different values of $\kappa_2$. We see that both the amplitude and the velocity of compressive IA solitary waves increase with deviation of cold electrons from the Maxwellian behavior (*via* decrease of $\kappa_1$). Furthermore, the spherical IA solitary waves move faster as compared to the corresponding cylindrical IA solitary waves. For $\nu = 0$, these results are in agreement with Ref. [18].

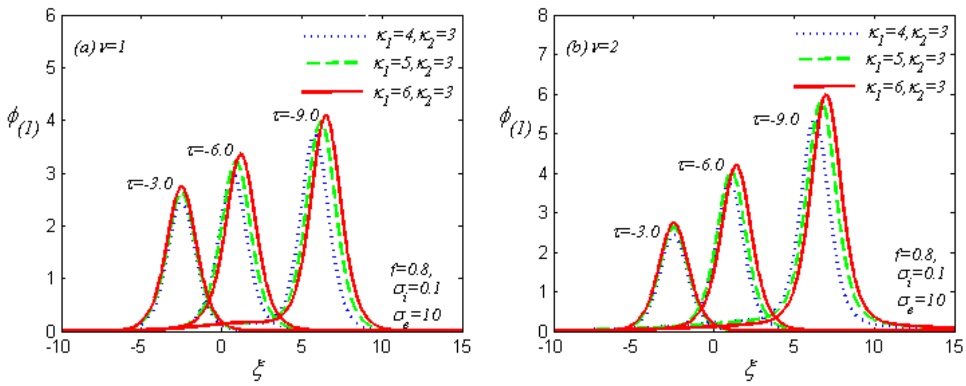


Fig. 4 – Numerical solution of Eq. (17) for several values of $\tau$ ranging from −9 to −3, and different values of $\kappa_1$; a) In cylindrical ( $\nu = 1$ ) geometry; b) in spherical ( $\nu = 2$ ) geometry. The other parameters are $\kappa_2 = 3$, $\sigma_e = 10$, $\sigma_i = 0.1$ and $f = 0.8$.

## 5. CONCLUSIONS

In the present work we have investigated the small amplitude solitons supported by nonplanar plasmas with two-temperature kappa-distributed electrons, as found in the Saturn's magnetosphere. The main results are as follows:

(*i*) Both compressive and rarefactive solitary structures are found to exist in the present model. It is clear that the polarity of IA solitons is sensitive to the interplay between the hot and cold electron populations. The occurrence of the rarefactive solitons is more probable for high values of the hot electron density. To explore the newly found nonplanar negative IA solitons, the experimental studies would need to have a second hot electron component for which $f = n_{c0} / n_{i0}$ being a few percent, with $\sigma_e = 10 - 30$ and $\sigma_i = 0.03 - 0.1$ [18, 51, 59]. In the other word, observation of such solitons in space plasmas would be suggestive of the presence of a secondary electron component satisfying such density and temperature values.

(*ii*) We found that when hot electrons evolve toward their thermodynamic equilibrium, the rarefacting IA solitons appear with smaller amplitude, while the compressive IA solitons show opposite behavior.

(*iii*) Also it is clear that the increase rate of the velocity and amplitude in the spherical geometry is faster than in cylindrical geometry.

Our theoretical results on the IA waves in the superthermal plasma can contribute to understanding localized electrostatic disturbances in space plasmas as well as in the laboratory plasmas, where kappa distributed two-temperature electron components are observed, such as in Saturn's magnetosphere [59]

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