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PROPERTIES OF THE AC DRIVEN FRENKEL-KONTOROVA MODEL WITH DEFORMABLE SUBSTRATE POTENTIAL*

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Abstract. Shapiro steps are studied in the ac driven overdamped Frenkel-Kontorova model with deformable substrate potential. Appearance of very large subharmonic steps due to deformation of the substrate potential significantly influences the stability and existence of harmonic steps. Strong correlation among harmonic and subharmonic steps has been observed in which, the larger the width of half-integer steps, the smaller that of harmonic steps. Deviation from the well-known Bessel-like oscillations of the steps with amplitude have been observed and three different types of behavior classified. Strong influence of the frequency of the ac driving force on the appearance and size of subharmonic steps has been observed.

Key words: Frenkel-Kontorova model, Shapiro steps, interference phenomena.

1. INTRODUCTION

Resonant solutions or Shapiro steps have been matter of many theoretical and experimental studies in systems such as charge-density wave conductors [1, 2] and systems of Josephson-junction arrays biased by external currents [3]. Numerous experimental and theoretical results in these systems particularly stimulated studies of the driven overdamped (dissipative) Frenkel-Kontorova (FK) model.

The standard Frenkel-Kontorova model represents a chain of harmonically interacting particles subjected to a sinusoidal substrate potential [4]. The FK model is characterized by the winding number which represents interparticle average distance or concentration. For rational values of winding number, the FK model has commensurate structure while for irrational values, the system is incommensurate. In the presence of an external dc+ac driving force, dynamics is characterized by the appearance of the staircase macroscopic response or the

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Shapiro steps in the response function $\bar{v}\left(\bar{F}\right)$ (the average velocity as a function of average driving force) of the system [5]. These steps are due to the dynamical mode-locking of the internal frequency that comes from the motion of particles over the periodic substrate potential with the frequency of an external ac force. When the locking appears at the integer values of the external frequency, the steps are called harmonic, while for the locking at noninteger rational values of the external frequency, they are called subharmonic. Although the standard FK model has been very successful in the explanations of many phenomena related to the Shapiro steps such as the amplitude or frequency dependence and the noise effects [6], it can not be used for the studies of subharmonic steps. It is well known that in the standard FK model, for commensurate structures with integer values of winding number only harmonic steps exist [5]. In the commensurate structures with noninteger values of winding number, subharmonic steps appear, however their size is too small which makes their analysis very difficult [5].

Contrary to the standard FK model, large subharmonic steps can appear in the presence of the deformable substrate potential, as it was shown in our previous work [7]. In the real physical situations, application of the standard FK model could be very restricted, and it is hard to believe that real physical systems could be "exactly" described by standard models or by employing perturbation methods. Introducing a family of nonlinear periodic deformable potentials, Remoissenet and Peyrard [8] obtained in a control manner by an adequate choice of parameters rich variety of deformable potentials related to the physical systems such as charge-density wave condensates, Josephson junctions, and crystals with dislocations.

We will examine the amplitude and frequency dependence of both harmonic and subharmonic steps in the presence of deformable substrate potential. In order to select the type of potential for our studies, our focus was on the models which refer to the same physical systems as the overdamped FK model, which are tunable, and reduce to the standard (sinusoidal) form in the case of zero deformations. The obtained results have shown strong correlation between harmonic steps and the appearance of large halfinteger steps. Deformation of the potential strongly influences the amplitude and frequency dependence of the Shapiro steps. As deformation increases, different types of amplitude and frequency dependence have been classified, and deviation from the well-known Bessel-like oscillations of the step size with amplitude has been observed.

2. MODEL

We consider the dissipative (overdamped) dynamics of series of coupled harmonics oscillators u_l subjected in one from the family of the parameterized deformable periodic potentials, the asymmetric deformable potential (ASDP) [8]:

$$V(u) = \frac{K}{2\pi} \frac{\left(1-r^2\right)^2 \left[1-\cos(2\pi u)\right]}{\left[1+r^2+2r\cos(\pi u)\right]^2}, \tag{1}$$

where K is the pinning strength and r is the shape parameter ($-1 < r < 1$). This potential can be tuned in a controlled manner from the sinusoidal (standard) for $r = 0$ that has been studied previously [5, 6] to an asymmetric periodic one for $0 < |r| < 1$ with a constant barrier height and two inequivalent successive wells. Precisely, here the asymmetry means that the pining in two successive potential minima is different. Though the potential wells are energetically equivalent, they are not physically (dynamically) equivalent since the pinning of particles strongly depends on the shape of potential well [8]. The system is driven by dc and ac forces: $F(t) = \bar{F} + F_{ac}\cos\left(2\pi\nu_0 t\right)$, where $\bar{F}$ is the dc force while F_{ac} and $2\pi\nu_0$ are the amplitude and frequency of the ac force, respectively. The equation of motion is

$$\dot{u}_l = u_{l+1} + u_{l-1} - 2u_l - V'\left(u_l\right) + F\left(t\right), \tag{2}$$

where $1 = -\frac{N}{2},...,\frac{N}{2}$. The ac force induces additional polarization energy into the system that is different from zero (less than zero) only when the velocity reaches the resonant values $\bar{v} = \left[\left(i\omega + j\right)/m\right]\nu_0$, where i, j and m are integers ($m = 1$ for harmonic, and $m > 1$ for subharmonic steps) [5]. The system will get locked since the average pinning energy of the locked state (on the step) is lower than that of the unlocked state. As $\bar{F}$ increases, the particles will stay locked until the pinning force can cancel the increase of $\bar{F}$. Equation 2 has been integrated using the fourth order Runge-Kutta method with the periodic boundary conditions for commensurate structure with the interparticle average distance, winding number $\omega = \frac{1}{2}$ (the case $\omega = \frac{1}{2}$ corresponds to structure with two particles per one potential well, while $\omega = 1$ would be one particle per one potential well). The average velocity $\bar{v}\left(\bar{F}\right)$, in particular, the amplitude and frequency dependence of the step size are analyzed for the different shapes of the substrate potential.

3. RESULTS

Deformable substrate potential for different values of the shape parameter, and the average velocity $\bar{v}\left(\bar{F}\right)$ for $r = 2$ are presented in Fig. 1.

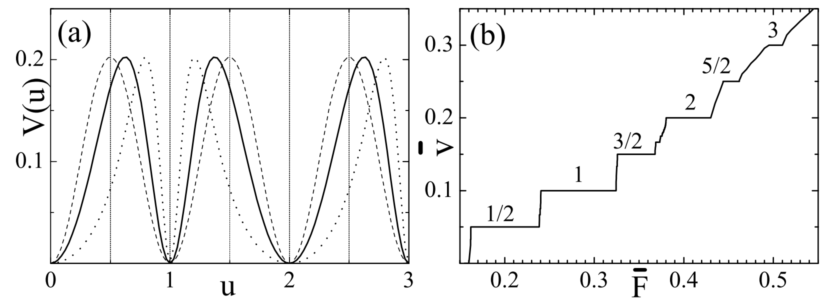

Fig. 1 – Asymmetric deformable potential in (a) for $K = 4$ and different values of the shape parameters $r = 0$, 0.2, and 0.5. Average velocity as a function of the average driving force in (b) for $\omega = 1/2$, $K = 4$, $F_{ac} = 0.2$, $\nu_0 = 0.2$, and $r = 0.2$. Numbers mark harmonic and halfinteger steps.

As we can see, when potential gets deformed, very large halfinteger and higher order subharmonic steps appear.

In Fig. 2, the amplitude dependence of the critical depinning force, the first harmonic and halfinteger (subharmonic) step is presented for different values of the shape parameter. Amplitude dependence for the standard case ($r = 0$) is presented by dashed line in Fig. 2a. It is well known that in the standard case, harmonic step width and the critical depinning force exhibit Bessel-like oscillations with the amplitude, where the maxima of one curve correspond to the minima of another.

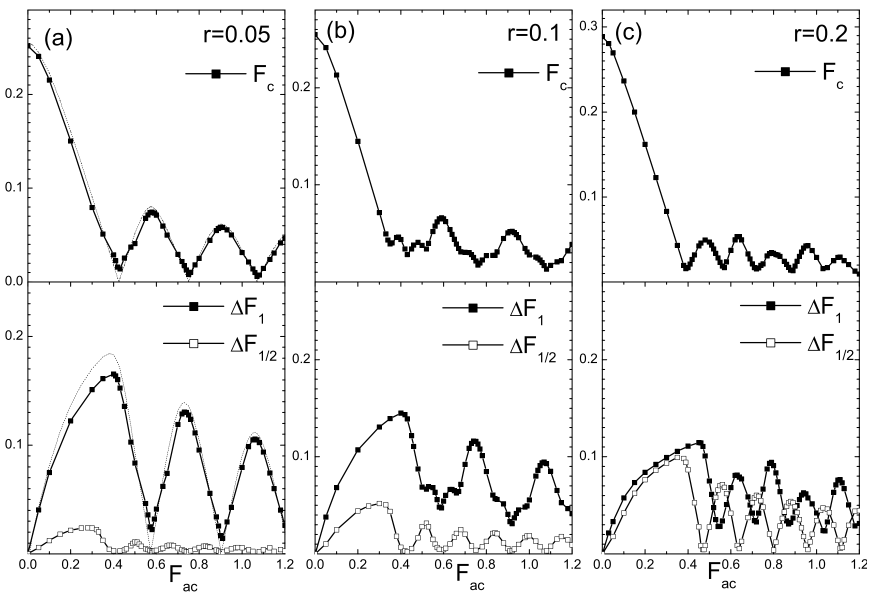

Fig. 2 – The width of the first harmonic ΔF_1, halfinteger step $\Delta F_{1/2}$, and the critical depinning force F_c as a function of the ac amplitude for $\omega = 1/2$, $K = 4$, $F_{ac} = 0.2$, $\nu_0 = 0.2$, and $r = 0.05$, 0.1, and 0.2 in (a–c) respectively. Dashed line in (a) corresponds to the case $r = 0$.

These oscillations appear due to back-forward jumps induced by the ac force. Namely, in the dc+ac driving systems, dynamics is characterized by the two types of motions: linear motion in the direction of the dc force and the back-forward jumps due to the ac force. During one period, particle at the site i, will jump n sites back, reach the $i-n$ site, and then hop again $n+1$ sites forward to the site $i+1$. In that way, by repeating these back-forward jumps with every period of the ac force it will move. The distance over which particle moves is determined by the amplitude of the ac force [2]. For the values of F_{ac} at the first maximum in Fig. 2a, during one period, particles will spend most of the time on the site and then hop to the next well, while for the values at the second maximum, it will jump one site back and two forward. As F_{ac} increases, particle will hop between more distant wells while spending less time on the sites, and consequently, the step width will decrease.

Deformation will affect this back-forward motion, and in our examination of the amplitude dependence, we could always classify three types of behavior:

- Standard behavior for small halfinteger steps when $r=0.05$ in Fig. 2a. The size of maxima decreases, and the width of harmonic step never goes completely to zero. However, the oscillations are still almost as in the case of pure sinusoidal standard potential $r=0$ where the maxima of ΔF_1 correspond to the minima of F_c;
- Behavior for intermediate halfinteger steps when $r=0.1$ in Fig. 2b. Two maxima of $\Delta F_{1/2}$ can be clearly seen per one period of ΔF_1 meanwhile the Bessel-like form of ΔF_1 and F_c is completely deformed and new maxima start to develop;
- Behavior in the presence of large halfinteger steps for $r=0.2$ in Fig. 2c. Harmonic step and critical depinning force oscillate, however, contrary to the standard case $r=0$, maxima of one curve correspond to the maxima of another while the anomalous oscillations (second maxima is lower than the third one) appear.

These three types of behavior were first observed and classified in the experimental studies of amplitude dependence of the Shapiro steps in the high-T_c grain-boundary junctions [9]. Dissipative dynamics of the Frenkel-Kontorova model is closely related to the many phenomena observed in the Josephson-junction arrays and the results in Fig. 2 are in good agreement with those experiments.

When potential is deformed, there are two groups of particles and two types of wells in which they can move. For the odd maxima of the curve for ΔF_1 in

Fig. 2c, particle from sharp (wide) minima will jump back to the same type of minima and then forward to the wide (sharp) minima (for the first maximum particles form sharp minima jump forward to wide minima and vice versa). However, for the second and all even maxima in Fig. 2c, particles in the sharp minima will jump backward to wide minima and from there forward to the wide minima again while those in wide minima will jump back to the sharp minima and from there forward to sharp minima. Due to different pinning, the type of minima between which particle jumps will affect their motion and therefore the step size and the maxima of the oscillations. This anomalous amplitude dependence as in Fig. 2c has been also observed in two-dimensional Josephson-junction arrays where deviation from the Bessel-like behavior and reduction of second lobe are result of the field-induced vortex super lattice and the broken symmetry, and they can not be obtained in single-junction case [10].

Contrary to the amplitude dependence, frequency dependence of Shapiro steps has been a matter of many controversies. In the charge-density wave systems, according to the classical approach [11], the step width and the critical depinning force should, after the initial increase, decrease to zero at the high frequencies. In contrast, according to simple single coordinate model motivated by the tunneling theory [2], the maximum step width and the magnitude of the fundamental component of the effective pinning force are independent of frequency at the high frequencies. In the systems with Josephson-junction arrays, according to the single junction model [12], the widths of harmonic steps remain frequency independent at the high frequencies. On the other side, in the systems with many degrees of freedom [9], disappearance of steps at the high frequencies has been observed (single junction models do not work well if the system is disordered).

In the FK model which represents an overdamped classical many-body model, and as in other systems with many degrees of freedom, the steps will remain strongly frequency dependent and disappear at the high frequencies. However, it was shown in our previous works that step can even oscillate with frequency [6]. In the high amplitude limit, when $\frac{F_{ac}}{F_{c0}} > 1$ (F_{c0} is dynamical dc threshold or the critical depinning force when the ac force is zero), the oscillations of the step width appear at the low frequencies. As in the case of the amplitude dependence, these oscillations will be affected by the deformation of the substrate potential and the appearance of halfinteger steps. In Fig. 3, the frequency dependence of the critical depinning force, the first harmonic, and half-integer steps is presented for different values of the shape parameter.

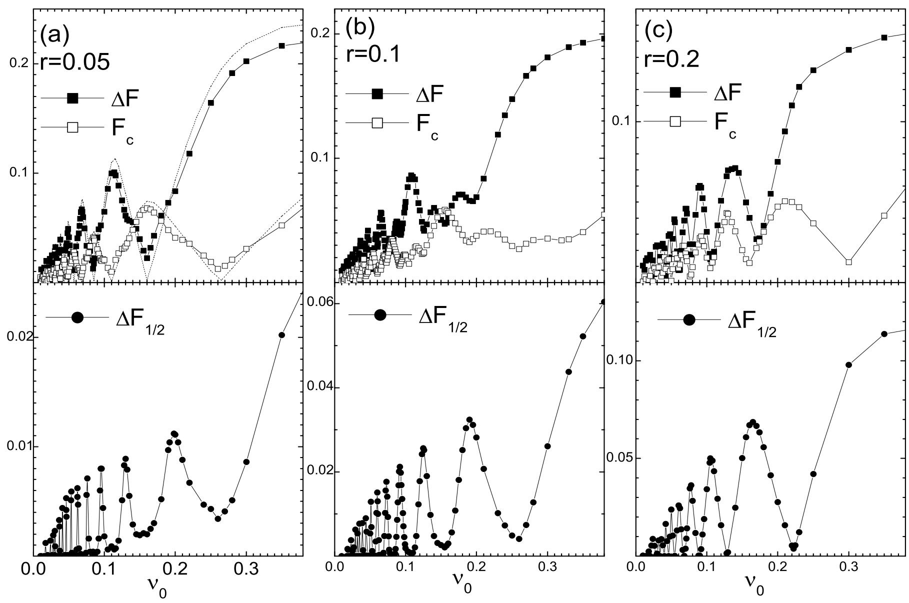

Fig. 3 – The width of the first harmonic ΔF_1, halfinteger step $\Delta F_{1/2}$, and the critical depinning force F_c as a function of the ac frequency for $\omega = 1/2$, $K = 4$, $F_{ac} = 0.5$, and $r = 0.05$, 0.1, and 0.2 in (a–c) respectively. Dashed line in (a) corresponds to the case $r = 0$.

These results clearly show how the frequency dependence evolves as the size of halfinteger step increases with the deformation. For the small deformation $r = 0.05$ and very small subharmonic steps in Fig. 3a, the behavior of the system is still like in the standard FK model [6], where the maxima (minima) of the oscillations for the step size correspond to the minima (maxima) of the critical depinning force. As the deformation increases for $r = 0.1$ in Fig. 3b, and large half-integer steps start to appear at the minima of the critical depinning force, the form of oscillations starts to change, and the new lobes in curves for ΔF and F_c to develop. For $r = 0.2$ and the very large half-integer step (they even oversize the harmonic step) in Fig. 3c, the behavior of ΔF_1 and F_c is completely changed; now the maxima of one curve corresponds to the maxima of another. However, the increase of deformation will also affect the dynamical dc threshold F_{c0} and therefore, the ratio $\frac{F_{ac}}{F_{c0}}$. Since F_{c0} starts to increase, at some point, for a given ac amplitude, the system will transfer from the high-amplitude limit $\frac{F_{ac}}{F_{c0}} > 1$ to the low-amplitude limit $\frac{F_{ac}}{F_{c0}} < 1$ where this oscillatory behavior disappears.

The physical origins of these oscillations can be understood, and an analogy with the amplitude dependence revealed if the results in Fig. 3 are expressed as a function of period $\frac{1}{\nu_0}$. In Fig. 4, the critical depinning force, the first harmonic, and halfinteger steps are presented as a function of period $\frac{1}{\nu_0}$ for different values of the shape parameter.

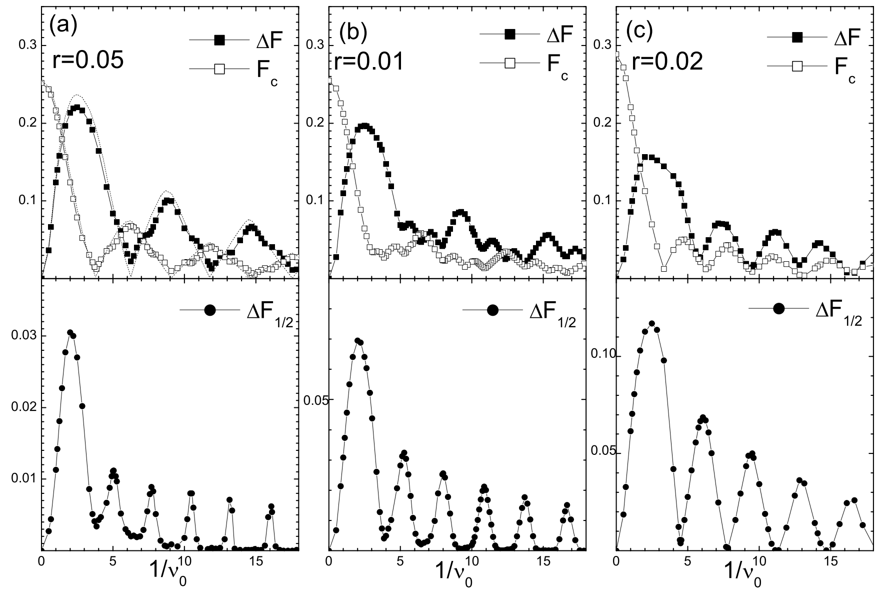

Fig. 4 – The width of the first harmonic ΔF_1, halfinteger step $\Delta F_{1/2}$, and the critical depinning force F_c as a function of period for $\omega = 1/2$, $K = 4$, $F_{ac} = 0.5$, and $r = 0.05$, 0.1, and 0.2 in (a–c) respectively. Dashed line in (a) corresponds to the case $r = 0$.

As in the case of amplitude dependence the oscillations appear due to the backward and forward motions of particles induced by the ac force. The distance over which particle moves is determined not only by the ac amplitude but also by the period or frequency. Particles will jump between more distant sites not only if the amplitude is large enough but also if they have enough time, which means if the period is long enough [6].

4. CONCLUSION

The above results have shown the existence of the strong correlation between harmonic and halfinteger steps where the size of halfinteger steps determines the properties and behavior of harmonic steps and the critical depinning force. The great analogy between our results for amplitude dependence in Fig. 2 and the results for frequency dependence in Fig. 4 as well as those experiments [9] clearly proves that the amplitude and the frequency play similar role in the ac-driven dynamics.

Presented results could be important for studies of charge- or spin-density waves systems and systems of Josephson-junction arrays that are particularly motivated by technical applications of the interference phenomena [1, 5]. Any fabrication of synchronization and superconducting Shapiro step devices requires a good theoretical guideline for the observation of Shapiro steps and these results bring new insight into the theory of interference phenomena.

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