Romanian Reports in Physics 72, 120 (2020)

DYNAMICS OF A CHARGED PARTICLE IN ELECTROMAGNETIC FIELD WITH JOULE EFFECT

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Received March 9, 2020

Abstract. In this paper, we present a new approach for solving equations of motion for the dynamics of charged particles moving under the action of electromagnetic field along with additional influence of the Joule effect.A new type of solving procedure is implemented here for the equations of motion of the charged particle, determined by the Lorentz force, and additionally taking into account the Joule effect. Meanwhile, the system of equations of motion has been successfully explored with respect to the existence of an analytical way for the presentation of the solution. Last but not least, we obtain the solutions in a form of a spiral-type motion. As a main result of this study, the equations of motion are reduced to a system of two nonlinear ordinary differential equations of first order (with regard to time t) for two unknown functions: 1) $w(t)$ (angular velocity of spiral rotation) and 2) $\xi(t)$ (spiral factor of motion for a charged particle). Moreover, the approximated solutions have been also obtained under appropriate simplifying assumptions.

Key words: Lorentz force, charged particle, Joule effect.

1. INTRODUCTION, EQUATIONS OF MOTION

The Lorentz force [1] plays a significant, crucial role at describing the dynamics of a charged particle, moving under the action of electro-magnetic field (including dynamics of solar wind near the magnetic shield of the planets); this is an extremely non-linear problem even in non-relativistic case insofar.

It is worth to note that only a few cases of analytical or semi-analytical solutions are known in the history of electro-magnetic theory, including trivial case of zero electric field along with constant magnetic field (which means a circular motion of a particle) as well as elegant Alfven's solution for solar wind in MHD theory [2].

The problem of motion of a charged particle under a Lorentz force has been studied in the past under various versions (*e.g.* Störmer's problem, the magnetic-binary problem, as well as in various models of Celestial Mechanics, Geophysical Sciences, and Plasma Physics) and there is a rich and extended international bibliography. In the current research, we will restrict ourselves in presenting a new analytical technique for solving equations of motion of a charged particle under the action of the Lorentz force with additional taking into consideration the Joule effect, which corresponds to the case of motion of a charged particle with additional influence of energy dissipation phenomenon or the heating effect during the particle's motion along its trajectory. As for the complete introduction to the problem under consideration, we recommend seminal works [3–4], where a significant historical retrospection has been made as well as all the difficulties regarding stability of motion are considered insofar (we mean the appropriate remarks regarding the stability of the solutions and how we are able to study it in the suggested particular problem).

Let us also preliminary note that the quantities that appear in Lorentz's equation of motion of a charged particle, such as the velocity of light, the electric charge of the particle, etc, are physical quantities. Therefore, before any mathematical treatment, these quantities, as well as the other fundamental physical quantities appearing in the equations of motion must be normalized in order to become dimensionless for their further considering as the parameters of the particular problem.

According to results reported in [3] [see formula (2)], the dynamics of a charged particle could be presented in the Cartesian coordinate system $\vec{r} = \{x, y, z\}$ by the equation of motion below (at given initial conditions):

$$\left(\frac{d\,\vec{v}}{d\,t}\right) = \vec{v} \times \vec{B} + \vec{E} - (\vec{v} \cdot \vec{E})\vec{v}, \tag{1.1}$$

where $\vec{v}$ is velocity of the particle, $\vec{v} = \{U, V, W\}$; charge q of particle equals to 1 (per unit of mass m); $\vec{B} = \vec{B}(\vec{r}, \mathrm{t})$ is the magnetic field, $\vec{B} = \{B_{\mathrm{i}}\}$; $\vec{E} = \vec{E}(\vec{r}, t)$ is the electric field, $\vec{E} = \{E_{\mathrm{i}}\}$, $i = \{x, y, z\}$. We consider a non-relativistic case here (as a first approximation).

The first two terms in the right hand part of Eq. (1.1) are the counterparts of the classical electromagnetic Lorentz force [3], whereas the third term corresponds to the Joule effect in the equation of motion (1.1) [4].

We should especially note that the physical nature of Lorentz force, along with the heating Joule effect, do not depend on the mass factor in the equation of motion (1.1).

2. SOLVING ODE SYSTEM FOR TIME-DEPENDENT COMPONENTS OF VELOCITY FIELD

Let us find a proper way for solving equations (1.1), taking into account a non-zero Joule effect in the aforementioned system (1.1). We will suggest the most obvious approach for obtaining such aforesaid solution as pointed below. The system of equations (1.1) could be presented accordingly:

$$\begin{cases} \dfrac{dU}{dt} = V \cdot B_z - W \cdot B_y + E_x - (E_x \cdot U + E_y \cdot V + E_z \cdot W) \cdot U, \\ \dfrac{dV}{dt} = W \cdot B_x - U \cdot B_z + E_y - (E_x \cdot U + E_y \cdot V + E_z \cdot W) \cdot V, \\ \dfrac{dW}{dt} = U \cdot B_y - V \cdot B_x + E_z - (E_x \cdot U + E_y \cdot V + E_z \cdot W) \cdot W, \end{cases} \tag{2.1}$$

According to the ansatz, which was used earlier in [5–11], let us initiate dot product of both the sides of (1.1) onto the velocity vector $\vec{v}$ = {U, V, W}; in this case, the system (2.1) should yield

$$\frac{1}{2}\frac{d(U^2 + V^2 + W^2)}{dt} = (E_x \cdot U + E_y \cdot V + E_z \cdot W) \cdot \left\{1 - (U^2 + V^2 + W^2)\right\}. \tag{2.2}$$

Let us note that expression $(E_x \cdot U + E_y \cdot V + E_z \cdot W)$ in Eq. (2.2) could be represented in other form

$$\begin{aligned}(E_x \cdot U + E_y \cdot V + E_z \cdot W) = \left|\vec{E}\right| \cdot \left|\vec{v}\right| \cdot \cos\alpha = \\ = \cos\alpha \cdot \sqrt{E_x^2 + E_y^2 + E_z^2} \cdot \sqrt{U^2 + V^2 + W^2},\end{aligned} \tag{*}$$

where α is the angle between vectors of electric field $\vec{E}$ and velocity $\vec{v}$ (cos α ≠ 0). Taking into account (*), we obtain from equation (2.2):

$$\frac{1}{2}\frac{d(U^2 + V^2 + W^2)}{dt} = \cos\alpha \cdot \sqrt{E_x^2 + E_y^2 + E_z^2} \cdot \sqrt{U^2 + V^2 + W^2} \cdot \{1 - (U^2 + V^2 + W^2)\} \Rightarrow$$

$$\Rightarrow \int \frac{d(U^2 + V^2 + W^2)}{\sqrt{U^2 + V^2 + W^2} \cdot \{1 - (U^2 + V^2 + W^2)\}} = 2\int \left(\cos\alpha \cdot \sqrt{E_x^2 + E_y^2 + E_z^2}\right) dt.$$

The left hand side of equation above could be then transformed to the proper analytical expression [12–13] with regard to function $(U^2 + V^2 + W^2)$ as below:

$$\begin{aligned} &2\int \frac{d(\sqrt{U^2 + V^2 + W^2})}{\{1 - (\sqrt{U^2 + V^2 + W^2})^2\}} = 2\int \left(\cos\alpha \cdot \sqrt{E_x^2 + E_y^2 + E_z^2}\right) dt \Rightarrow \\ &\Rightarrow \frac{1}{2}\ln\left|\frac{1 + \sqrt{U^2 + V^2 + W^2}}{1 - \sqrt{U^2 + V^2 + W^2}}\right| = \int \left(\cos\alpha \cdot \sqrt{E_x^2 + E_y^2 + E_z^2}\right) dt \Rightarrow \\ &\Rightarrow U^2 + V^2 + W^2 = \left(\frac{\exp\left(2\int \left(\cos\alpha \cdot \sqrt{E_x^2 + E_y^2 + E_z^2}\right) dt\right) - 1}{\exp\left(2\int \left(\cos\alpha \cdot \sqrt{E_x^2 + E_y^2 + E_z^2}\right) dt\right) + 1} \right)^2. \end{aligned} \tag{2.3a}$$

The obvious restriction should be valid for the type (2.3a) of the partial solutions, namely $\sqrt{U^2 + V^2 + W^2} < 1$.

Let us also note that both sides of Eq. (2.2) should be satisfied, if we choose the special condition below for the solutions of system (2.1) to be valid

$$(U^2 + V^2 + W^2) = 1 . \tag{2.3b}$$

Taking into account (2.3b), the system of Eqs. (2.1) is to be reduced accordingly

$$\begin{cases} \dfrac{dU}{dt} = V \cdot B_z - \sqrt{1-(U^2+V^2)} \cdot B_y + E_x - \left(E_x \cdot U + E_y \cdot V + E_z \cdot \sqrt{1-(U^2+V^2)}\right) \cdot U, \\ \dfrac{dV}{dt} = \sqrt{1-(U^2+V^2)} \cdot B_x - U \cdot B_z + E_y - \left(E_x \cdot U + E_y \cdot V + E_z \cdot \sqrt{1-(U^2+V^2)}\right) \cdot V, \\ (U^2+V^2+W^2) = 1 \Rightarrow W = \sqrt{1-(U^2+V^2)}, \end{cases} \tag{2.4}$$

so, we have obtained the system (2.4) of two non-linear differential equations of first order (with regard to time t) for two unknown functions *U* and *V*.

3. SPIRAL-TYPE SOLUTIONS OF EQS. (1.1)–(2.1) FOR THE VELOCITY FIELD {*U*, *V*, *W*}

Let us assume that the velocity field $\vec{v}$ = {*U*, *V*, *W*} of the particle is to be associated with spiral-type motion in general form (3.1) (Fig. 1):

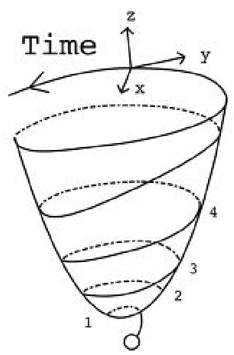

Fig. 1 – Type of spiral motion.

As per assumption above, it means that the components of velocity field *U*, *V*, *W* should be presented as below:

$$U = \xi(t) \cdot \cos(w(t) \cdot t), \quad V = \xi(t) \cdot \sin(w(t) \cdot t), \quad W = W(t), \tag{3.1}$$

where $w(t)$ is the function, associated with the proper angular velocity of spiral rotation; $\xi\,(t)$ is a spiral factor. For example:

1) If $\xi(t) = a \cdot t + c$, $W(t) = b \cdot t$, we should obtain the spiral of screw line type.

2) If $\xi(t) = a \cdot \exp(b \cdot t)$, $W(t) = c \cdot t$, we should obtain the 3-D logarithmic spiral, here $\{a, b, c\}$ are supposed to be arbitrary positive real constants.

Thus if we substitute the representation (3.1) in the components of solution in Eqs. (2.4), we obtain as below

$$\begin{cases} \dfrac{d(\xi\cdot\cos(w\cdot t))}{dt} = \xi\cdot\sin(w\cdot t)\cdot B_z - \sqrt{1-\xi^2}\cdot B_y + E_x - \\ \qquad - (E_x\cdot\xi\cdot\cos(w\cdot t) + E_y\cdot\xi\cdot\sin(w\cdot t) + E_z\cdot\sqrt{1-\xi^2})\cdot\xi\cdot\cos(w\cdot t), \\ \dfrac{d(\xi\cdot\sin(w\cdot t))}{dt} = \sqrt{1-\xi^2}\cdot B_x - \xi\cdot\cos(w\cdot t)\cdot B_z + E_y - \\ \qquad - (E_x\cdot\xi\cdot\cos(w\cdot t) + E_y\cdot\xi\cdot\sin(w\cdot t) + E_z\cdot\sqrt{1-\xi^2})\cdot\xi\cdot\sin(w\cdot t), \\ (U^2 + V^2 + W^2) = 1 \quad \Rightarrow \quad W = \sqrt{1-\xi^2}, \end{cases} \tag{3.2}$$

so, we have obtained the system (3.2) of two non-linear differential equations of first order (with regard to time t) for two unknown functions: 1) the spiral factor $\xi(t)$ and 2) the factor of angular velocity of spiral rotation $w(t)$.

For further solving of the system (3.2) above, we should transform it accordingly as below:

$$\begin{cases} \xi'\cdot\cos(w\cdot t) - \xi\cdot\sin(w\cdot t)\cdot(w'\cdot t + w) = \xi\cdot\sin(w\cdot t)\cdot B_z - \sqrt{1-\xi^2}\cdot B_y + E_x - \\ \qquad - (E_x\cdot\xi\cdot\cos(w\cdot t) + E_y\cdot\xi\cdot\sin(w\cdot t) + E_z\cdot\sqrt{1-\xi^2})\cdot\xi\cdot\cos(w\cdot t), \\ \xi'\cdot\sin(w\cdot t) + \xi\cdot\cos(w\cdot t)\cdot(w'\cdot t + w) = \sqrt{1-\xi^2}\cdot B_x - \xi\cdot\cos(w\cdot t)\cdot B_z + E_y - \\ \qquad - (E_x\cdot\xi\cdot\cos(w\cdot t) + E_y\cdot\xi\cdot\sin(w\cdot t) + E_z\cdot\sqrt{1-\xi^2})\cdot\xi\cdot\sin(w\cdot t). \end{cases} \tag{3.3}$$

Furthermore, let us multiply the first equation of (3.3) by $\sin(w{\cdot}t)$, the second by $\cos(w \cdot t)$, then let us linearly combine them (one to each other) properly:

$$\xi\cdot w' = \frac{\sin(w\cdot t)}{t}\cdot\left(\sqrt{1-\xi^2}\cdot B_y - E_x\right) + \frac{\cos(w\cdot t)}{t}\cdot\left(\sqrt{1-\xi^2}\cdot B_x - E_y\right) - \xi\cdot\left(\frac{B_z + w}{t}\right) \tag{3.4}$$

To obtain the second linear combination of equations (3.3), let us divide the first equation of (3.3) by $\sin(w \cdot t)$, the second by $\cos(w \cdot t)$, then let us linearly combine them (one to each other) in appropriate way

$$\xi' = \sin(w\cdot t)\cdot\left(\sqrt{1-\xi^2}\cdot B_x - E_y\right) - \cos(w\cdot t)\cdot\left(\sqrt{1-\xi^2}\cdot B_y - E_x\right) - \\ -(E_x\cdot\xi\cdot\cos(w\cdot t) + E_y\cdot\xi\cdot\sin(w\cdot t) + E_z\cdot\sqrt{1-\xi^2})\cdot\xi. \tag{3.5}$$

Thus, equations (3.4)–(3.5) present the ultimate system of two non-linear differential equations of first order (with regard to time t) for two unknown functions, $\xi(t)$ and $w(t)$:

$$\begin{cases} \dfrac{d\xi}{dt} = \sin(w\cdot t)\cdot\left(\sqrt{1-\xi^2}\cdot B_x + E_y\right) - \cos(w\cdot t)\cdot\left(\sqrt{1-\xi^2}\cdot B_y - E_x\right) - \\ \quad -(E_x\cdot\xi\cdot\cos(w\cdot t) + E_y\cdot\xi\cdot\sin(w\cdot t) + E_z\cdot\sqrt{1-\xi^2})\cdot\xi, \\ \xi\dfrac{dw}{dt} = \dfrac{\sin(w\cdot t)}{t}\cdot\left(\sqrt{1-\xi^2}\cdot B_y - E_x\right) + \dfrac{\cos(w\cdot t)}{t}\cdot\left(\sqrt{1-\xi^2}\cdot B_x + E_y\right) - \xi\cdot\left(\dfrac{B_z + w}{t}\right). \end{cases} \tag{3.6}$$

4. APPROXIMATED SOLUTION FOR SPIRAL-TYPE VELOCITY FIELD {*U*, *V*, *W*}

Obviously, the aforementioned system of equations (3.6) is a system of two non-linear ordinary differential equations of the first order which could be solved by numerical methods only [12].

Nevertheless, let us suggest a kind of approximated solutions for Eqs. (3.6): if we take into account the set of additional assumptions $\xi \ll 1$, $w \ll 1$ in (3.6) for the time interval $t > t_0$ (besides, $w(t)$ is supposed to be a slowly varying function depending on t), the aforesaid assumptions should simplify Eqs. (3.6) by the series of Taylor expansions as below (where we should neglect further at the right hand parts of equations the terms of second order or less ~ $\{\xi^2, w^2, \xi\cdot w\}$)

$$\begin{cases} \dfrac{d\xi}{dt} \cong w\cdot t\cdot\left((1-\dfrac{1}{2}\xi^2)\cdot B_x + E_y\right) - (1-\dfrac{(w\cdot t)^2}{2})\cdot\left((1-\dfrac{1}{2}\xi^2)\cdot B_y - E_x\right) - \\ \quad -(E_x\cdot\xi\cdot(1-\dfrac{(w\cdot t)^2}{2}) + E_y\cdot\xi\cdot w\cdot t + E_z\cdot(1-\dfrac{1}{2}\xi^2))\cdot\xi, \qquad \Rightarrow \\ \xi\dfrac{dw}{dt} \cong w\left((1-\dfrac{1}{2}\xi^2)B_y - E_x\right) + \dfrac{(1-\dfrac{(wt)^2}{2})}{t}\left((1-\dfrac{1}{2}\xi^2)B_x + E_y\right) - \xi\left(\dfrac{B_z + w}{t}\right) \end{cases} \tag{4.1}$$

$$\begin{cases} \dfrac{d\xi}{dt} \cong w\cdot t\cdot(B_x + E_y) - B_y + E_x - E_z\cdot\xi, \\ \xi\cdot t\cdot\dfrac{dw}{dt} \cong w\cdot t\cdot(B_y - E_x) + B_x + E_y - \xi\cdot B_z. \end{cases}$$

The next step in simplification of system (4.1) stems from our previous assumption that $w(t)$ is supposed to be a slowly varying function depending on time t; indeed, in this case the appropriate approximation of second equation of (4.1) yields ($B_y \neq E_x$)

$$w\cdot(B_y - E_x) + \frac{B_x + E_y - \xi\cdot B_z}{t} \cong \xi\cdot\left(\frac{dw}{dt}\right) \to 0 \Rightarrow$$
$$\Rightarrow w\cdot(B_y - E_x) \cong \frac{\xi\cdot B_z - B_x - E_y}{t}. \tag{4.2}$$

It means that we should choose $B_x = -E_y$ in (4.2). So, we obtain from the first of equations (4.1)

$$\frac{d\xi}{dt} \cong -B_y + E_x - E_z\cdot\xi, \quad \Rightarrow \quad \frac{d\xi}{dt} + \Lambda\cdot\zeta \cong Y, \quad \{\Lambda = E_z, \quad Y = E_x - B_y\}$$
$$\xi \cong \exp\left(-\int\Lambda dt\right)\cdot\left[\int(Y\cdot\exp\left(\int\Lambda dt\right))dt + \xi_0\right] \tag{4.3}$$
$$\{\xi_0 = \text{const.}\}$$

For example, if $\{\Lambda, Y\}$ = const., (4.3) yields as below

$$\xi \cong \frac{Y}{\Lambda} + \xi_0\cdot\exp(-\Lambda\cdot t), \tag{4.4}$$

where we should choose constants $\{\Lambda, Y, \xi_0\}$ in such a way that $\xi << 1$ in (4.3)–(4.4) and $w << 1$ in (4.2).

5. FINAL PRESENTATION OF THE SOLUTION FOR VELOCITY FIELD OF SPIRAL-TYPE

Let us present the partial approximated solution for velocity field of spiral type (3.1) for the motion of charged particle (1.1) ($B_y \neq E_x$, $B_x = -E_y$) in its final form:

$$U = \xi(t)\cdot\cos(w\cdot t), \quad V = \xi(t)\cdot\sin(w\cdot t), \quad W = \sqrt{1-\xi^2}, \quad \xi^2 \ll 1, \tag{5.1}$$

where w is the angular velocity of spiral rotation

$$w\cdot\left(B_y - E_x\right) \cong \frac{\xi\cdot B_z}{t} \tag{5.2}$$

whereas the spiral factor ξ (*t*) in the expressions (5.1)–(5.2) is given by

$$\xi \cong \exp\left(-\int \Lambda dt\right)\cdot\left[\int (Y\cdot\exp\left(\int \Lambda dt\right))dt+\xi_0\right] \tag{5.3}$$
$$\left\{\xi_0=\text{const},\quad \Lambda=E_z,\quad Y=E_x-B_y\right\}.$$

Let us schematically plot in Fig. 2 the approximation (5.1)–(5.3) for the component $U(t)$ of velocity field (5.1) (depending on time t), which corresponds to the case (4.4) of constant components of the electromagnetic field $\vec{B}$ = $\{B_i\}$, $\vec{E}$ = $\{E_i\}$ (in Fig. 1 we designate $x = t$ just for the aim of presenting the plot of solution).

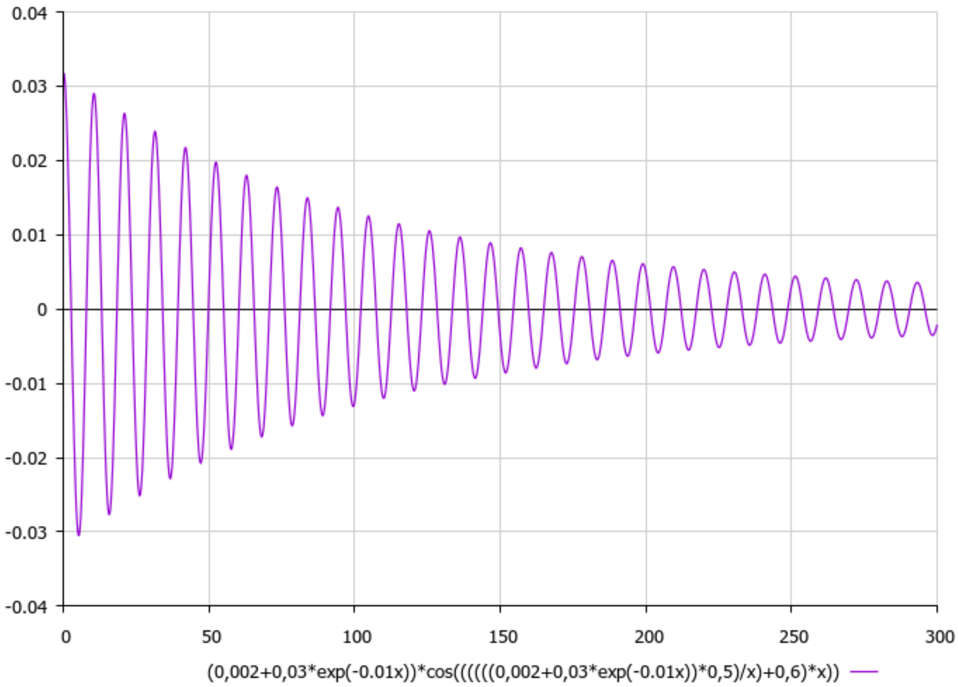

Fig. 2 – A schematic plot for the component $U(t)$ of the velocity field (5.1), which corresponds to the case (4.4) of the constant electromagnetic field.

6. DISCUSSION

As we can see from the derivation above, equations of motion (for the dynamics of charged particles moving under the action of electro-magnetic field along with the additional influence of Joule effect) are proved to be very hard to solve analytically. Nevertheless, as a first step we have succeeded in obtaining the special differential invariant (2.2) of the system (2.1), which stems from (1.1) and can be associated with the dynamics of transformation (invariant (2.3a)) or even local conservation (invariant (2.3b)) of kinetic energy of the particle during its motion in electromagnetic field under the action of dissipative Joule effect.

We pointed out two classes of partial solutions, which correspond to the special differential invariant (2.2), namely (2.3a) and (2.3b). Furthermore, at the second step we suggest to obtain the solutions in the form (3.1) of spiral-type motion.

If we choose the special condition (2.3b) for the solutions of system (2.1) as to be valid, the equations of motion should be reduced to the form (3.6) in this case. Then, we have obtained the appropriate approximated solutions (4.2)–(4.3) for the function $w(t)$ (angular velocity of spiral rotation) and for the function $\xi\,(t)$ (spiral factor of motion for a charged particle), under additional simplifying assumptions $\xi \ll 1$ and $w \ll 1$ in (4.3)–(4.4). These assumptions restrict the choosing of components of electromagnetic field $\{B_i\}$, $\{E_i\}$, which take part in expressions for the approximated solutions (5.2)–(5.3) ($B_y \neq E_x$, $B_x = -E_y$).

If we additionally consider the type (2.3a) of partial solutions for which $\sqrt{U^2+V^2+W^2} < 1$, whereas the electric field $\vec{E} = \vec{E}(\vec{r},\, t)$ in Eqs. (1.1) is represented in the form $\{E_\mathrm{i}\} = \{0, 0, E_z\}$ (*i.e.* the vector of electric field coincides with the Oz-axis in the Cartesian coordinate system $\vec{r} = \{x, y, z\}$), the equations of motion should be reduced to the form (6.1) below. Indeed, in this case the angle α in (2.3a) is equal to $\alpha = \arccos(\xi/\sqrt{\xi^2 + W^2}) \cong \arccos(\xi/W)$, where ξ is defined in (3.1) or (5.1) (we consider the case $\xi \ll 1$).

Thus, if we substitute the representation (3.1) in the components of solution in equations (2.1), (2.3a), where $\{E_i\} = \{0, 0, E_z\}$, we obtain accordingly (by linearly combining them one to each other properly as in Eqs. (3.6)):

$$\begin{cases} \dfrac{d\xi}{dt} = \sin(w\cdot t)\cdot W\cdot B_x - \cos(w\cdot t)\cdot W\cdot B_y - E_z\cdot W\cdot \xi, \\ \xi\dfrac{dw}{dt} = \dfrac{\sin(w\cdot t)}{t}\cdot W\cdot B_y + \dfrac{\cos(w\cdot t)}{t}\cdot W\cdot B_x - \xi\cdot\left(\dfrac{B_z + w}{t}\right). \end{cases} \tag{6.1}$$

$$W \cong \left(\frac{\exp\left(2\int\left(\left(\dfrac{\xi}{W}\right)\cdot E_z\right)dt\right) - 1}{\exp\left(2\int\left(\left(\dfrac{\xi}{W}\right)\cdot E_z\right)dt\right) + 1} \right), \quad \xi \ll 1.$$

As we can see, the system of equations (6.1) is a system of two non-linear integro-differential equations which could be solved by numerical methods only (there are known very rare cases, when using the Laplace transform of integrals and derivatives, an integro-differential equation can be solved [12]).

Thus, we have fully solved the system of equations of motion for the dynamics of charged particles moving under the action of electromagnetic field along with the additional influence of Joule effect. Such new result (5.1)–(5.3) can be used in calculations of the laboratory experiments in plasma physics. We have suggested a new type of physically reasonable solution of spiral-type (having been applied to the problem under investigation), which has been explored here by mathematical methods with respect to their existence, but also these solutions should be then explored with regard to the reliability and convenience in using them at engineering calculations after completing the series of experiments with plasma in laboratory.

To the best of our knowledge, analytical solutions for the general case do not exist and the only way to get some kind of information about the intrinsic properties and behavior of the particular dynamical system is to calculate approximated solutions by using various numerical methods. So, any new theoretical method or approach for even the particular solving of the aforementioned system of equations would be useful on the level of practical applications.

7. CONCLUSION

As for the purpose of current research reported in this paper, we can formulate it as follows: the main aim is to find a kind of exact solution (of spiral type) to the system of equations under consideration. Namely, each exact solution can clarify the structure, intrinsic code and topology of the variety of possible solutions (from mathematical point of view).

In this challenging analytic survey, we present a new approach for solving equations of the dynamics of a charged particle (describing its motion in variable, time-dependent electromagnetic field) under additional taking into account the Joule effect. A new type of the solving procedure is implemented in the case of solving momentum equation for the aforementioned dynamics of a charged particle, determined by the Lorentz force along with the dissipative Joule effect, in the non-relativistic case. Meanwhile, the system of momentum equations has been successfully solved analytically.

The main result of the current research can be formulated as follows: the analytic algorithm is pointed out for solving momentum equation, which has been reduced to the analytical solution of three nonlinear ordinary differential equations with respect to the components of velocity of the particle. Moreover, new partial analytical solutions of spiral type have been obtained for a special case of electromagnetic field. Such new results, namely (5.1)–(5.3), and (6.1), can be used in laboratory experiments in plasma physics, albeit they can be applied in ordinary way for testing the accuracy of numerical methods.

Finally, a recent book [14] as well as articles [15–18] should be also referred to; they deal with some aspects of the problem under consideration in the current paper.

Acknowledgements. In this research, Dr. Sergey Ershkov is responsible for the general ansatz and the solving procedure, simple algebra manipulations, calculations, results of the article in Sections 1-5, and also is responsible for the search of approximated solutions. Dr. Dmytro Leshchenko is responsible for theoretical investigations as well as for the deep survey in literature on the problem under consideration. Both authors agreed with the results and conclusions of each other in Sections 1-7.

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