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ON FINDING CLOSED-FORM SOLUTIONS TO SOME NONLINEAR FRACTIONAL SYSTEMS *VIA* THE COMBINATION OF MULTI-LAPLACE TRANSFORM AND THE ADOMIAN DECOMPOSITION METHOD

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Abstract. In this article, two- and three-dimensional nonlinear fractional partial differential systems are solved by employing the methods of double and triple Laplace-Adomian decomposition. The applicability of these methods is examined on some known examples. The numerical simulations of the obtained results reveal that these methods are simple and effective on finding the exact and approximate solutions for some mathematical models. Moreover, they show that these methods are fully harmonious with the complexity of nonlinear problems.

Key words: Caputo fractional derivatives, multi-Laplace transform, inverse of multi-Laplace transform.

1. INTRODUCTION

Fractional calculus plays a significant role in modeling many physical and mathematical phenomena in biology, chemistry, fluid mechanics, psychology, and many other sciences; this is since these phenomena depend on the past time as their dependence on the instantaneous time [1–12]. The concept of fractional calculus was firstly introduced in 1695, and then developed by many researchers such as, Riemann, Liouville, Hadamard, and Caputo [13–19]. The definition of fractional calculus in the Caputo sense is very popular since it appears in many real-life applications.

Solving the fractional differential equations has got the attentions of many mathematicians; and consequently, numerous analytical and numerical methods have been found to obtain the exact and approximate solutions of them. Such of these methods are the Adomian decomposition method (ADM), homotopy perturbation method (HPM), variational iteration method (VIM), the Laplace transform method, homotopy analysis method (HAM), residual power series (RPS) method, and many other methods [20–47].

Recently, combinations of the above methods have been introduced to solve nonlinear convolution partial differential equations. One of these combinations, which is related to our study, is a combination of the double Laplace transform and the decomposition method, which solved some differential equations and integro-differential equations [48]. Later, the properties and convolution theorem for the double Laplace transform were studied in [49]. Very recently, the triple Laplace decomposition was successfully applied to solve the nonlinear coupled Burgers equation in [50].

The purpose of this study is to extend the application of multi Laplace decomposition method to derive the exact solution of some nonlinear time-fractional equations and systems about initial data. This paper is organized as follows. In Sec. 2, some preliminaries of fractional calculus and multi-Laplace transforms theory are given. In Sec. 3, the principle of the proposed method to solve nonlinear partial differential equations is discussed. In Sec. 4, three illustrative examples are solved by using double and triple Laplace transform algorithm based on decomposition method. Finally, the conclusion is given in Sec. 5.

2. FUNDAMENTAL CONCEPTS

In this Section, we recall the definitions of Caputo time-fractional derivative and multi-Laplace transform. Also, we present some of their fundamental properties.

Definition 2.1 *For $n \in \mathbb{N}$ and $t \geq 0$, the Caputo time-fractional derivative D_t^γ of a function $f(x,t)$ of order $\gamma > 0$ is defined by*

$$D_t^\gamma f(x,t) = \begin{cases} \frac{1}{\Gamma(n-\gamma)} \int_0^t (t-\tau)^{n-\gamma-1} \partial_t^n f(x,\tau) d\tau, & n-1 < \gamma \leq n, x \in I, \\ \partial_t^n f(x,\tau), & \gamma = n \in \mathbb{N}, x \in I. \end{cases} \tag{1}$$

Definition 2.2 *Let $f(x,t)$ be a piecewise continuous function defined on $\mathbb{R}^+ \times \mathbb{R}^+$. Then, the double Laplace transform of $f(x,t)$, denoted by $F(p,s)$, is defined by*

$$F(p,s) = \mathcal{L}_x \mathcal{L}_t [f(x,y,t)] = \int_0^\infty \int_0^\infty e^{-px-st} f(x,t) dt dx, \tag{2}$$

where p and s are complex variables.

Similarly, the triple Laplace transform of a function $f(x,y,t)$ is defined as follows:

Definition 2.3 *For a piecewise continuous function $f(x,y,t)$ defined on $\mathbb{R}^+ \times \mathbb{R}^+ \times$*

$\mathbb{R}^+$, *the triple Laplace transform of* $f(x,y,t)$, *denoted by* $F(p,q,s)$, *is defined by*

$$F(p,q,s) = \mathcal{L}_x\mathcal{L}_y\mathcal{L}_t[f(x,y,t)] = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-px-qy-st} f(x,y,t)dtdydx, \quad (3)$$

where p, q, *and* s *are complex variables.*

The inverse of triple Laplace transform of the function $F(p,q,s)$ is defined as follows:

$$\begin{aligned} f(x,y,t) &= \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}[F(p,q,s)] \\ &= \frac{1}{2\pi i}\int_{z_1-i\infty}^{z_1+i\infty} e^{px}dp \frac{1}{2\pi i}\int_{z_2-i\infty}^{z_2+i\infty} e^{qy}dq \frac{1}{2\pi i}\int_{z_3-i\infty}^{z_3+i\infty} e^{st}F(p,q,s)ds, \end{aligned} \quad (4)$$

where $z_1 = Re(p)$, $z_2 = Re(q)$ and $z_3 = Re(s)$.

Theorem 2.4 *Let* $n \in \mathbb{N}$ *and* $\gamma \in (n-1,n]$. *Assume that* $\partial_u^n f(x,y,t) \in (0,a) \times (0,b) \times (0,c)$ *for any* $a,b,c > 0, |f(x,y,t)| \leq w e^{xl_1+yl_2+tl_3}$ *and* $u = x,y$ *or* t. *Then, the triple Laplace transforms of Caputo time-fractional derivative of* $D_x^{\gamma}f(x,y,t)$, $D_y^{\gamma}f(x,y,t)$, *and* $D_t^{\gamma}f(x,y,t)$ *are defined as follows:*

$$\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t[D_x^{\gamma}f(x,y,t)] = p^{\gamma}F(p,q,s) - \sum_{k=0}^{n-1} p^{\gamma-k-1}\mathcal{L}_x\mathcal{L}_y\left(\partial_x^k f(0,y,t)\right), \quad (5)$$

$$\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t[D_y^{\gamma}f(x,y,t)] = q^{\gamma}F(p,q,s) - \sum_{j=0}^{n-1} q^{\gamma-j-1}\mathcal{L}_x\mathcal{L}_y\left(\partial_y^j f(x,0,t)\right), \quad (6)$$

and

$$\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t[D_t^{\gamma}f(x,y,t)] = s^{\gamma}F(p,q,s) - \sum_{i=0}^{n-1} s^{\gamma-i-1}\mathcal{L}_x\mathcal{L}_y\left(\partial_t^i f(x,y,0)\right). \quad (7)$$

3. GENERAL PRINCIPLE OF THE TRIPLE LAPLACE DECOMPOSITION METHOD

In this Section, we give and clarify the principle of the proposed methodology to obtain the exact solutions for fractional initial value problems (IVPs) by using the method of triple Laplace transform operator. The basic principle of our proposed method includes the following procedure.

Let us consider the following nonlinear nonhomogeneous fractional partial equation:

$$\begin{cases} D_t^{\gamma}f(x,y,t) + L(f(x,y,t)) + N(f(x,y,t)) = g(x,y,t) \\ f(x,y,0) = f_0(x,y), \ \ x \in \mathbb{R}, t \geq 0, \gamma \in (0,1], \end{cases} \quad (8)$$

where N represents the nonlinear differential operator, L is the linear operator, and $g(x,y,t)$ is the nonhomogeneous term.

First: We operate the triple Laplace transform on both sides of equation (8) and we get that

$$\begin{aligned} F(p,q,s) &= \frac{F_0(p,q)}{s} + \frac{G(p,q,s)}{s^\gamma} - \frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(L\left(f(x,y,t)\right)\right) \\ &- \frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(N\left(f(x,y,t)\right)\right). \end{aligned} \tag{9}$$

Second: We represent a solution as the following infinite decomposition series. For the linear term, we have

$$f\left(x,y,t\right) = \sum_{m=0}^{\infty} f_m(x,y,t). \tag{10}$$

For the nonlinear term, we decompose

$$N\left(f\left(x,y,t\right)\right) = \sum_{m=0}^{\infty} A_m(f\left(x,y,t\right)), \tag{11}$$

where A_m describe the Adomian polynomials [51] defined by

$$A_m = \frac{1}{m!}\frac{\partial^m}{\partial\beta^m}[N\left(\sum_{n=0}^{\infty}\beta^n f_n(x,y,t)\right)]_{\beta=0}, m = 0,1,2,\ldots \tag{12}$$

Now, we substitute (10) and (11) in (9) and we get that

$$\begin{aligned} \mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(\sum_{m=0}^{\infty} f_m(x,y,t)\right) &= \frac{F_0\left(p,q\right)}{s} + \frac{G\left(p,q,s\right)}{s^\gamma} - \frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(\sum_{m=0}^{\infty} A_m\right) \\ &- \frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(L\left(\sum_{m=0}^{\infty} f_m(x,y,t)\right)\right). \end{aligned} \tag{13}$$

Third: We compare both sides of equation (13) and we deduce the following:

$$\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(f_0(x,y,t)\right) = \frac{F_0\left(p,q\right)}{s} + \frac{G\left(p,q,s\right)}{s^\gamma}, \tag{14}$$

$$\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(f_1(x,y,t)\right) = -\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(L\left(f_0(x,y,t)\right)\right) - \frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(A_0\right), \tag{15}$$

$$\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(f_2(x,y,t)\right) = -\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(L\left(f_1(x,y,t)\right)\right) - \frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(A_1\right), \tag{16}$$

and so on.

Finally: We apply the inverse triple Laplace transform operator on both sides of step three and we reach that

$$f_0(x,y,t) = f_0\left(x,y\right) + \mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{G\left(p,q,s\right)}{s^\gamma}\right) \tag{17}$$

$$f_{m+1}(x,y,t) = -\mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\tfrac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(L\left(f_m(x,y,t)\right)\right)\right)$$
$$-\mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\tfrac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(A_m\right)\right),\ m \geq 0. \tag{18}$$

4. NUMERICAL EXAMPLES

In this Section, we illustrate the superiority and potentiality of the proposed method for solving three time-fractional systems with suitable initial conditions [51]. All calculations have been carried out manually.

Example 1 *Consider the following IVPs:*

$$\begin{cases} D_t^{2\gamma} f = f_x^2 - f^2, \\ f(x,0) = 0\ , \\ D_t^\gamma f(x,0) = e^x, \end{cases} \tag{19}$$

where $x \in \mathbb{R}\ ,\ t \geq 0,\ \text{and}\ \gamma \in (0,1]$. *Here, we notice that for* $\gamma = 1$, *the exact solution of (19) is* $f(x,t) = te^x$.

As we stated in the previous Section, the double Laplace equation of the time-fractional equation (19) using the initial data has the following form

$$F(p,s) = \frac{1}{s^{\gamma+1}}\mathcal{L}_x(e^x) + \frac{1}{s^{2\gamma}}\mathcal{L}_x\mathcal{L}_t\left(f_x^2 - f^2\right). \tag{20}$$

Applying the inverse double Laplace transform operator on both sides of (20), we have

$$f(x,t) = \frac{t^\gamma e^x}{\Gamma(\gamma+1)} + \mathcal{L}_x{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{1}{s^{2\gamma}}\mathcal{L}_x\mathcal{L}_t\left(f_x^2 - f^2\right)\right). \tag{21}$$

The Laplace-Adomian decomposition method represents the solution $f(x,t)$ *using this infinite series*

$$f(x,t) = \sum_{m=0}^{\infty} f_m(x,t). \tag{22}$$

Substituting $f(x,t)$ *of (22) into (21), we conclude that*

$$\sum_{m=0}^{\infty} f_m(x,t) = \frac{t^\gamma e^x}{\Gamma(\gamma+1)} + \mathcal{L}_x{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{1}{s^{2\gamma}}\mathcal{L}_x\mathcal{L}_t\left(\sum_{m=0}^{\infty} B_m - \sum_{m=0}^{\infty} A_m\right)\right), \tag{23}$$

where B_m *and* A_m *are the Adomian polynomial represented by (12). Hence*

$$f_x^2 = \sum_{m=0}^{\infty} B_m \tag{24}$$

and

$$f^2=\sum_{m=0}^{\infty}A_m\ . \tag{25}$$

The first few terms of the Adomian polynomial are given by

$$B_0=f_{0x}^2, B_1=2f_{1x}f_{0x},\ldots,B_m=\sum_{j=0}^{\mathrm{m}}f_{jx}f_{(m-j)x} \tag{26}$$

$$A_0=f_0^2, A_1=2f_1f_0,\ldots,A_m=\sum_{i=0}^{\mathrm{m}}f_if_{(m-i)}\ . \tag{27}$$

By comparing both sides of equation (23), we get

$$f_0(x,t)=\frac{t^{\gamma}e^{x}}{\Gamma(\gamma+1)} \tag{28}$$

$$f_{m+1}(x,t)=\mathcal{L}_x{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{1}{s^{2\gamma}}\mathcal{L}_x\mathcal{L}_t(B_m-A_m)\right)\ with\ m\geq 0. \tag{29}$$

Thus,

$$\begin{aligned}f_1(x,t) &= \mathcal{L}_x{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{1}{s^{2\gamma}}\mathcal{L}_x\mathcal{L}_t(B_0-A_0)\right)\\ &= \mathcal{L}_x{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{1}{s^{2\gamma}}\mathcal{L}_x\mathcal{L}_t\left(f_{0x}^2-f_0^2\right)\right)\\ &= \mathcal{L}_x{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{1}{s^{2\gamma}}\mathcal{L}_x\mathcal{L}_t\left(\frac{t^{\gamma}e^{x}}{\Gamma(\gamma+1)}-\frac{t^{\gamma}e^{x}}{\Gamma(\gamma+1)}\right)\right)=0. \end{aligned}\tag{30}$$

Therefore, the solution is given by

$$f(x,t)=\sum_{m=0}^{\infty}f_m(x,t)=f_0(x,t)+f_1(x,t)+f_2(x,t)+\cdots=\frac{t^{\gamma}e^{x}}{\Gamma(\gamma+1)}. \tag{31}$$

It is worth mentioning that when we take $\gamma=1$ *in (31), we get the same results that were obtained in [52].*

In Fig. 1, we plot the profile solutions to (31) for different values of the time-fractional order γ. It can be seen that for $t<1$ and when γ increases, the value of the function f decreases. While as, the situation is completely converse for $t>1$. Moreover, the profile solutions are continuously mapping to the integer-derivative case upon increasing the fractional order. We may say that the fractional-derivative possesses a memory index that preserves some inherited properties.

Example 2 *Consider the system of nonlinear nonhomogeneous fractional partial*

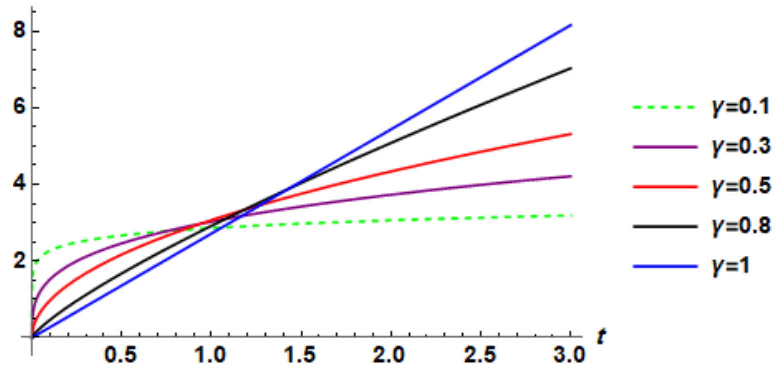

Fig. 1 – Profile solutions to the fractional IVP given in (31) for different values of the time-fractional order.

differential equations. For $x, y \in \mathbb{R}\,,\ t \geq 0,\ \gamma \in (0,1]\,,$

$$\begin{cases} D_t^\gamma f - g_x h_y = 1, \\ D_t^\gamma g - h_x f_y = 5, \\ D_t^\gamma h - f_x g_y = 5, \end{cases} \tag{32}$$

with initial data

$$\begin{cases} f(x,y,0) = x + 2y, \\ g(x,y,0) = x - 2y, \\ h(x,y,0) = -x + 2y. \end{cases}$$

The triple Laplace equation of the time-fractional system (32) using the initial data has the following form:

$$F(p,q,s) = \frac{1}{s}\mathcal{L}_x\mathcal{L}_y(x+2y) + \frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(1) + \frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(g_x h_y) \tag{33}$$

$$G(p,q,s) = \frac{1}{s}\mathcal{L}_x\mathcal{L}_y(x-2y) + \frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(5) + \frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(h_x f_y) \tag{34}$$

$$H(p,q,s) = \frac{1}{s}\mathcal{L}_x\mathcal{L}_y(-x+2y) + \frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(5) + \frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(f_x g_y). \tag{35}$$

Applying the inverse triple Laplace transform operator on both sides of (33), (34) and (35) we have

$$f(x,y,t) = x + 2y + \frac{t^\gamma}{\Gamma(\gamma+1)} + \mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(g_x h_y)\right) \tag{36}$$

$$g(x,y,t) = x - 2y + \frac{5t^\gamma}{\Gamma(\gamma+1)} + \mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(h_x f_y)\right) \tag{37}$$

$$h(x,y,t) = -x+2y+\frac{5t^{\gamma}}{\Gamma(\gamma+1)}+{\mathcal{L}_x}^{-1}{\mathcal{L}_y}^{-1}{\mathcal{L}_t}^{-1}\left(\frac{1}{s^{\gamma}}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(f_x g_y)\right). \quad (38)$$

Using the above recursive relations about the Adomian polynomial, we obtain

$$f_0(x,y,t) = x+2y+\frac{t^{\gamma}}{\Gamma(\gamma+1)}; \quad (39)$$

$$f_{m+1}(x,t) = {\mathcal{L}_x}^{-1}{\mathcal{L}_y}^{-1}{\mathcal{L}_t}^{-1}\left(\frac{1}{s^{\gamma}}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(C_m)\right),\ m \geq 0; \quad (40)$$

$$g_0(x,y,t) = x-2y+\frac{5t^{\gamma}}{\Gamma(\gamma+1)}; \quad (41)$$

$$g_{m+1}(x,t) = {\mathcal{L}_x}^{-1}{\mathcal{L}_y}^{-1}{\mathcal{L}_t}^{-1}\left(\frac{1}{s^{\gamma}}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(D_m)\right),\ m \geq 0; \quad (42)$$

$$h_0(x,y,t) = -x+2y+\frac{5t^{\gamma}}{\Gamma(\gamma+1)}; \quad (43)$$

$$h_{m+1}(x,t) = {\mathcal{L}_x}^{-1}{\mathcal{L}_y}^{-1}{\mathcal{L}_t}^{-1}\left(\frac{1}{s^{\gamma}}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(E_m)\right),\ m \geq 0. \quad (44)$$

The first few terms of the Adomian polynomial of C_m , D_m , E_m *are given by*

$$C_0 = g_{0x}h_{0y}\ , C_1 = g_{0x}h_{1y}+g_{1x}h_{0y}, \ldots, C_m = \sum_{j=0}^{\mathrm{m}} g_{jx}h_{(m-j)y}, \quad (45)$$

$$D_0 = h_{0x}f_{0y}\ , D_1 = h_{0x}f_{1y}+h_{1x}f_{0y}, \ldots, D_m = \sum_{j=0}^{\mathrm{m}} h_{jx}f_{(m-j)y}\ , \quad (46)$$

$$E_0 = f_{0x}g_{0y}\ , E_1 = f_{0x}g_{1y}+f_{1x}g_{0y}, \ldots, E_m = \sum_{j=0}^{\mathrm{m}} f_{jx}g_{(m-j)y}\ . \quad (47)$$

Now, we obtain the other components by using (40), (42), and (44). In fact, we get

$$\begin{aligned} f_1(x,y,t)) &= {\mathcal{L}_x}^{-1}{\mathcal{L}_y}^{-1}{\mathcal{L}_t}^{-1}\left(\frac{1}{s^{\gamma}}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(C_0)\right) \\ &= {\mathcal{L}_x}^{-1}{\mathcal{L}_y}^{-1}{\mathcal{L}_t}^{-1}\left(\frac{1}{s^{\gamma}}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(g_{0x}h_{0y}\ \right)\right) \\ &= {\mathcal{L}_x}^{-1}{\mathcal{L}_y}^{-1}{\mathcal{L}_t}^{-1}\left(\frac{1}{s^{\gamma}}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(1*2)\right) = \frac{2t^{\gamma}}{\Gamma(\gamma+1)}. \quad (48) \end{aligned}$$

$$
\begin{aligned}
g_1(x,y,t) &= \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(D_0)\right)\\
&= \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(h_{0x}f_{0y}\right)\right)\\
&= \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(-1*2)\right)=\frac{-2t^\gamma}{\Gamma(\gamma+1)}. \qquad (49)
\end{aligned}
$$

$$
\begin{aligned}
h_1(x,y,t) &= \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(E_0)\right)\\
&= \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(f_{0x}g_{0y}\right)\right)\\
&= \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(1*-2)\right)=\frac{-2t^\gamma}{\Gamma(\gamma+1)}. \qquad (50)
\end{aligned}
$$

$$
\begin{aligned}
f_2(x,y,t) &= \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(C_1)\right)\\
&= \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(g_{0x}h_{1y}+g_{1x}h_{0y}\right)\right)\\
&= \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(1*2+1*-2)\right)=0. \qquad (51)
\end{aligned}
$$

$$
\begin{aligned}
g_2(x,y,t) &= \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(D_1)\right)\\
&= \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(h_{0x}f_{1y}+h_{1x}f_{0y}\right)\right)=0. \qquad (52)
\end{aligned}
$$

$$
\begin{aligned}
h_2(x,y,t) &= \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(E_1)\right)\\
&= \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(f_{0x}g_{1y}+f_{1x}g_{0y}\right)\right)=0. \qquad (53)
\end{aligned}
$$

Therefore, the solution of the above nonlinear nonhomogeneous fractional system is given by

$$f(x,y,t)=x+2y+\frac{3t^\gamma}{\Gamma(\gamma+1)}, \qquad (54)$$

$$g(x,y,t)=x-2y+\frac{3t^\gamma}{\Gamma(\gamma+1)}, \qquad (55)$$

$$h(x,y,t)=-x+2y+\frac{3t^\gamma}{\Gamma(\gamma+1)}. \qquad (56)$$

Note that, for $\gamma = 1$, the solution of (32) is

$$\begin{cases} f(x,y,t) = x + 2y + 3t \\ g(x,y,t) = x - 2y + 3t \\ h(x,y,t) = -x + 2y + 3t, \end{cases}$$

which is the exact solution obtained in [52].

Example 3 *Consider the system of nonlinear fractional partial differential equations, for $x, y \in \mathbb{R}$, $t \geq 0$, $\gamma \in (0,1]$:*

$$\begin{cases} D_t^\gamma f + g_x h_y - g_y h_x = -f, \\ D_t^\gamma g + h_x f_y + h_y f_x = g, \\ D_t^\gamma h + f_x g_y + f_y g_x = h; \end{cases} \tag{57}$$

with initial conditions

$$\begin{cases} f(x,y,0) = e^{x+y}, \\ g(x,y,0) = e^{x-y}, \\ h(x,y,0) = e^{-x+y}. \end{cases}$$

By using the triple Laplace transform of the time-fractional equation (57) with the given initial conditions, we get

$$F(p,q,s) = \frac{1}{s}\mathcal{L}_x\mathcal{L}_y\left(e^{x+y}\right) - \frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(f) + \frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(g_y h_x - g_x h_y\right); \tag{58}$$

$$G(p,q,s) = \frac{1}{s}\mathcal{L}_x\mathcal{L}_y\left(e^{x-y}\right) + \frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(g) - \frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(h_x f_y + h_y f_x\right); \tag{59}$$

$$H(p,q,s) = \frac{1}{s}\mathcal{L}_x\mathcal{L}_y\left(e^{-x+y}\right) + \frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(h) - \frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(f_x g_y + f_y g_x\right). \tag{60}$$

Then by employing the formulas (5)-(7) on both sides of (58)-(60), respectively, we deduce

$$\begin{aligned} f(x,y,t) &= e^{x+y} - \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left(\tfrac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(f)\right) \\ &+ \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left(\tfrac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(g_y h_x - g_x h_y\right)\right), \end{aligned} \tag{61}$$

$$\begin{aligned} g(x,y,t) &= e^{x-y} + \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left(\tfrac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(g)\right) \\ &- \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left(\tfrac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(h_x f_y + h_y f_x\right)\right), \end{aligned} \tag{62}$$

$$\begin{aligned} h(x,y,t) &= e^{-x+y} + \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left(\tfrac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(h)\right) \\ &- \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left(\tfrac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(h_x f_y + h_y f_x\right)\right). \end{aligned} \tag{63}$$

Using the above recursive relations about the Adomian polynomial, we obtain

$$f_0(x,y,t) = e^{x+y} \tag{64}$$

$$\begin{aligned} f_{m+1}(x,t) &= -\mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\tfrac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(f_m)\right) \\ &+ \mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\tfrac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(V_m-U_m)\right),\ m\geq 0 \end{aligned} \tag{65}$$

$$g_0(x,y,t)=e^{x-y} \tag{66}$$

$$\begin{aligned} g_{m+1}(x,t) &= \mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\tfrac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(g_m)\right) \\ &- \mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\tfrac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(K_m+R_m)\right),\ m\geq 0 \end{aligned} \tag{67}$$

and

$$h_0(x,y,t)=e^{-x+y} \tag{68}$$

$$\begin{aligned} h_{m+1}(x,t) &= \mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\tfrac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(h_m)\right) \\ &- \mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\tfrac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(S_m+P_m)\right),\ m\geq 0, \end{aligned} \tag{69}$$

where the m^{th} terms of the Adomian polynomials of V_m ,U_m ,K_m , R_m ,S_m, and P_m are given by

$$\begin{array}{l} V_m=\sum_{j=0}^{\mathrm{m}} g_{jy}h_{(m-j)x} \\ U_m=\sum_{j=0}^{\mathrm{m}} g_{jx}h_{(m-j)y} \\ K_m=\sum_{j=0}^{\mathrm{m}} h_{jx}f_{(m-j)y} \\ R_m=\sum_{j=0}^{\mathrm{m}} h_{jy}f_{(m-j)x} \\ S_m=\sum_{j=0}^{\mathrm{m}} f_{jx}g_{(m-j)y} \\ P_m=\sum_{j=0}^{\mathrm{m}} f_{jy}g_{(m-j)x}. \end{array} \tag{70}$$

By using (64)-(70), we get that the other components are the following:

$$\begin{aligned} f_1(x,y,t) &= -\mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(f_0)\right) \\ &+ \mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(V_0-U_0)\right) \\ &= -\mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(e^{x+y}\right)\right) \\ &+ \mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(g_{0y}h_{0x}-g_{0x}h_{0y}\right)\right) \\ &= \frac{-t^\gamma e^{x+y}}{\Gamma(\gamma+1)}, \end{aligned} \tag{71}$$

$$\begin{aligned} g_1(x,y,t) &= \mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(e^{x-y}\right)\right) \\ &- \mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(h_{0x}f_{0y}+h_{0y}f_{0x}\right)\right) \\ &= \frac{t^\gamma e^{x-y}}{\Gamma(\gamma+1)}, \end{aligned} \tag{72}$$

$$\begin{aligned} h_1(x,y,t) &= \mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(e^{-x+y}\right)\right) \\ &- \mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(f_{0x}g_{0y}+f_{0y}g_{0x}\right)\right) \\ &= \frac{t^\gamma e^{-x+y}}{\Gamma(\gamma+1)}, \end{aligned} \tag{73}$$

$$\begin{aligned} f_2(x,y,t) &= -\mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(f_1)\right) \\ &+ \mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(V_1-U_1)\right) \\ &= \mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(\frac{t^\gamma e^{x+y}}{\Gamma(\gamma+1)}\right)\right) \\ &+ \mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(g_{0y}h_{1x}+g_{1y}h_{0x}\right)\right) \\ &- \mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\frac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\left(g_{0x}h_{1y}+g_{1x}h_{0y}\right)\right) \\ &= \frac{t^{2\gamma} e^{x+y}}{\Gamma(2\gamma+1)}, \end{aligned} \tag{74}$$

$$\begin{aligned} g_2(x,y,t) &= \mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\tfrac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(g_1)\right) \\ &- \mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\tfrac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(K_1-R_1)\right) \\ &= \tfrac{t^{2\gamma} e^{x-y}}{\Gamma(2\gamma+1)}, \end{aligned} \tag{75}$$

$$\begin{aligned} h_2(x,y,t) &= \mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\tfrac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(h_1)\right) \\ &- \mathcal{L}_x{}^{-1}\mathcal{L}_y{}^{-1}\mathcal{L}_t{}^{-1}\left(\tfrac{1}{s^\gamma}\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t(S_1-P_1)\right) \\ &= \tfrac{t^{2\gamma} e^{-x+y}}{\Gamma(2\gamma+1)}. \end{aligned} \tag{76}$$

By finding the coefficient functions, we obtain that the solution of the above system is

$$f(x,y,t) = e^{x+y}\left(\sum_{j=0}^{\infty} \frac{(-1)^j t^{j\gamma}}{\Gamma(j\gamma+1)}\right), \tag{77}$$

$$g(x,y,t) = e^{x-y}\left(\sum_{j=0}^{\infty} \frac{t^{j\gamma}}{\Gamma(j\gamma+1)}\right) = e^{x-y} E_{\gamma}(t), \tag{78}$$

$$\begin{aligned} h(x,y,t) &= e^{-x+y}\left(\sum_{j=0}^{\infty} \frac{t^{j\gamma}}{\Gamma(j\gamma+1)}\right) \\ &= e^{-x+y} E_{\gamma}(t), \end{aligned} \tag{79}$$

where E_{γ} is the Mittag-Leffler function. We point out here that for $\gamma = 1$, the exact solution of (57) is

$$\begin{cases} f(x,y,t) = e^{x+y-t}, \\ g(x,y,t) = e^{x-y+t}, \\ h(x,y,t) = e^{-x+y+t}. \end{cases}$$

Moreover, we may consider the following functions as n^{th}-approximate solutions as alternatives to $f,\ g,\ h$, respectively.

$$f_n(x,y,t) = e^{x+y}\left(\sum_{j=0}^{\mathrm{n}} \frac{(-1)^j t^{j\gamma}}{\Gamma(j\gamma+1)}\right), \tag{80}$$

$$g_n(x,y,t) = e^{x-y}\left(\sum_{j=0}^{\mathrm{n}} \frac{t^{j\gamma}}{\Gamma(j\gamma+1)}\right), \tag{81}$$

$$h_n(x,y,t) = e^{-x+y}\left(\sum_{j=0}^{\mathrm{n}} \frac{t^{j\gamma}}{\Gamma(j\gamma+1)}\right). \tag{82}$$

The profile solutions of the 10^{th}-approximate functions $f_n,\ g_n,\ h_n$ for different values of γ are shown in Fig. 2. We observe that the solutions bifurcate for small values of the fractional-derivative as in the case of $\gamma = 0.1$. On the other side, the effect of the fractional-derivative acting on the propagations of the functions $f_n,\ g_n,\ h_n$ is given in Fig. 3.

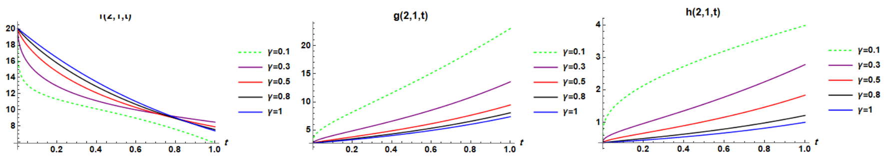

Fig. 2 – Profile solutions to the 10^{th}-order approximate solutions of the functions $f,\ g,\ h$ for different values of the time-fractional order.

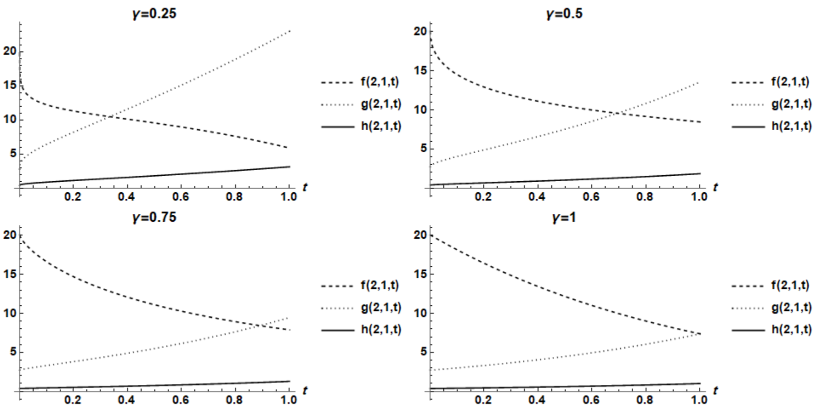

Fig. 3 – The propagations of the functions $f_{10},\ g_{10},\ h_{10}$ for different values of the time-fractional order.

5. CONCLUSION

In this work, we solved analytically some time-fractional systems by extending the multi-Laplace Adomian decomposition method. This method was employed to solve three different examples in the Laplace space, and we obtained the same results as used by other techniques. The suggested method is robust and is effective in extracting closed-form solutions for different classes of linear and nonlinear fractional problems arising in physical sciences.

REFERENCES

1. R.L. Magin, C. Ingo, L. Colon-Perez, W. Triplett, and T.H. Mareci, Characterization of anomalous diffusion in porous biological tissues using fractional order derivatives and entropy, Microporous and Mesoporous Materials **178**, 39-43 (2013).
2. S. Zhang and H.Q. Zhang, Fractional sub-equation method and its applications to nonlinear fractional PDEs, Phys. Lett. A **375**(7), 1069-1073 (2011).
3. F. Mainardi, M. Raberto, R. Gorenflo, and E. Scalas, Fractional calculus and continuous-time finance II: the waiting-time distribution, Physica A: Statistical Mechanics and its Applications **287**(3-4), 468-481 (2000).
4. M. Caputo, Linear models of dissipation whose Q is almost frequency independent-II, Geophysical Journal International **13**(5), 529-539 (1967).
5. M. Caputo and M. Fabrizio, A new definition of fractional derivative without singular kernel, Progr. Fract. Differ. Appl. **1** (2), 73-85 (2015).
6. J.A. Tenreiro Machado, M.F. Silva, R.S. Barbosa, I.S. Jesus, C.M. Reis, M.G. Marcos, and A.F. Galhano, Some applications of fractional calculus in Engineering, Math. Probl. Eng. **2010**, 639801 (2010).
7. D. Baleanu, A. Jajarmi, and J.H. Asad, Classical and fractional aspects of two coupled pendulums, Rom. Rep. Phys. **71**, 103 (2019).

8. M.A. Abdelkawy, An improved collocation technique for distributed-order fractional partial differential equations, Rom. Rep. Phys. **72**, 104 (2020).
9. F. Bekhouche, M. Alquran, and I. Komashynska, Explicit rational solutions for time-space fractional nonlinear equation describing the propagation of bidirectional waves in low-pass electrical lines, Rom. J. Phys. **66**, 114 (2021).
10. D. Baleanu, S. Sadat Sajjadi, A. Jajarmi, O. Defterli, and J.H. Asad, The fractional dynamics of a linear triatomic molecule, Rom. Rep. Phys. **73**, 105 (2021).
11. I. Jaradat, M. Alquran, Q. Katatbeh, F. Yousef, S. Momani, and D. Baleanu, An avant-garde handling of temporal-spatial fractional physical models, Int. J. Nonlinear Sci. Numer. Simul. **21**(2), 183-194 (2020).
12. F. Yousef, M. Alquran, I. Jaradat, S. Momani, and D. Baleanu, New fractional analytical study of three-dimensional evolution equation equipped with three memory indices, J. Comput. Nonlinear Dynam. **14**(11), 111008 (2019).
13. L. Torkzadeh, Numerical behavior of nonlinear Duffing equations with fractional damping, Rom. Rep. Phys. **73**, 113 (2021).
14. K.B. Oldham and J. Spanier, *The fractional calculus*, Academic Press, New York and London, 1974.
15. K.B. Oldham, A new approach to the solution of electrochemical problems involving diffusion, Anal. Chem. **41**, 1904-1905 (1969).
16. R.L. Magin, Fractional calculus models of complex dynamics in biological tissues, Comput. Math. Appl. **59**(5), 1586-1593 (2010).
17. F. Mainardi, Fractional calculus and waves in linear viscoelasticity, Imperial College Press, London, UK (2010).
18. R.L. Bagley and P.J. Torvik, Fractional calculus in the transient analysis of viscoelastically damped structures, AIAA J. **23**(6), 918-925 (1985).
19. A. Atangana and D. Baleanu, New fractional derivatives with non-local and non-singular kernel: theory and application to heat transfer model, Thermal. Sci. **20**, 763-769 (2016).
20. Q. Wang, Numerical solutions for fractional KdV-Burgers equation by Adomian decomposition method, Applied Mathematics and Computation **182**(2), 1048-1055 (2006).
21. S. Momani and Z. Odibat, Comparison between the homotopy perturbation method and the variational iteration method for linear fractional partial differential equations, Computers and Mathematics with Applications **54**(7-8), 910-919 (2007).
22. J.H. He, Approximate analytical solution for seepage flow with fractional derivatives porous media, Computer Methods in Applied Mechanics and Engineering **167**(1-2), 57-68 (1998).
23. S. Das, Analytical solution of a fractional diffusion equation by variational iteration method, Computers and Mathematics with Applications **57**(3), 483-487 (2009).
24. M. Ali, M. Alquran, I. Jaradat, N. Abu Afouna, and D. Baleanu, Dynamics of integer-fractional time-derivative for the new two-mode Kuramoto-Sivashinsky model, Rom. Rep. Phys. **72**, 103 (2020).
25. M. Alquran, I. Jaradat, D. Baleanu, and M. Syam, The Duffing model endowed with fractional time derivative and multiple pantograph time delays, Rom. J. Phys. **64**, 107 (2019).
26. M. Alquran and I. Jaradat, Delay-asymptotic solutions for the time-fractional delay-type wave equation, Physica A **527**, 121275 (2019).
27. M. Alquran and I. Jaradat, A novel scheme for solving Caputo time-fractional nonlinear equations: theory and application, Nonlinear Dyn. **91**(4), 2389-2395 (2018).
28. M. Al-Smadi, O. Abu Arqub, and S. Momani, Numerical computations of coupled fractional resonant Schrödinger equations arising in quantum mechanics under conformable fractional derivative

sense, Phys. Scr. **95**(7), 075218 (2020).
29. S. Momani, O. Abu Arqub, A. Freihat, and M. Al-Smadi, Analytical approximations for Fokker-Planck equations of fractional order in multistep schemes, Appl. Comput. Math. **15**(3), 319-330 (2016).
30. M. Al-Smadi, O. Abu Arqub, and S. Hadid, An attractive analytical technique for coupled system of fractional partial differential equations in shallow water waves with conformable derivative, Commun. Theor. Phys. **72**(8), 085001 (2020).
31. M. Al-Smadi, O. Abu Arqub, and S. Hadid, Approximate solutions of nonlinear fractional Kundu-Eckhaus and coupled fractional massive Thirring equations emerging in quantum field theory using conformable residual power series method, Phys. Scr. **95**(10), 105205 (2020).
32. M. Alquran, I. Jaradat, D. Baleanu, and R. Abdel-Muhsen, An analytical study of $(2+1)$-dimensional physical models embedded entirely in fractal space, Rom. J. Phys. **64**, 103 (2019).
33. M. Alquran, I. Jaradat, S. Momani, and D. Baleanu, Chaotic and solitonic solutions for a new time-fractional two-mode Korteweg-de Vries equation, Rom. Rep. Phys. **72**, 117 (2020).
34. M. Ali, M. Alquran, and I. Jaradat, Explicit and approximate solutions for the Conformable-Caputo time-fractional diffusive predator–prey model, Int. J. Appl. Comput. Math. **7**, 90 (2021).
35. M. Alquran, M. Alsukhour, M. Ali, and I. Jaradat, Combination of Laplace transform and residual power series techniques to solve autonomous n-dimensional fractional nonlinear systems, Nonlinear Engineering **10**(1), 282-292 (2021).
36. M. Alquran, F. Yousef, F. Alquran, T.A. Sulaiman, and A. Yusuf, Dual-wave solutions for the quadratic-cubic Conformable-Caputo time-fractional Klein-Fock-Gordon equation, Mathematics and Computers in Simulation **185**, 62-76 (2021).
37. I. Jaradat, M. Alquran, and M. Al-Dolat, Analytic solution of homogeneous time-invariant fractional IVP, Adv. Differ. Equ. **2018**, 143 (2018).
38. I. Jaradat, M. Alquran, S. Sivasundaram, and D. Baleanu, Simulating the joint impact of temporal and spatial memory indices *via* a novel analytical scheme, Nonlinear Dyn. **103**(3), 2509-2524 (2021).
39. I. Jaradat, M. Alquran, and R. Abdel-Muhsen, An analytical framework of 2D diffusion, wave-like, telegraph, and Burgers' models with twofold Caputo derivatives ordering, Nonlinear Dyn. **93**(4), 1911-1922 (2018).
40. I. Jaradat, M. Alquran, and K. Al-Khaled, An analytical study of physical models with inherited temporal and spatial memory, Eur. Phys. J. Plus. **133**, 162 (2018).
41. F. Yousef, M. Alquran, I. Jaradat, S. Momani, and D. Baleanu, Ternary-fractional differential transform schema: theory and application, Adv. Diff. Eqs. **2019**, 197 (2019).
42. I. Jaradat, M. Alquran, F. Yousef, S. Momani, and D. Baleanu, On $(2+1)$-dimensional physical models endowed with decoupled spatial and temporal memory indices, Eur. Phys. J. Plus **134**, 360 (2019).
43. M. Al-Smadi and O. Abu Arqub, Computational algorithm for solving fredholm time-fractional partial integro-differential equations of dirichlet functions type with error estimates, Appl. Math. Comput. **342**, 280-294 (2019).
44. M. Al-Smadi, O. Abu Arqub, and S. Momani, A computational method for two-point boundary value problems of fourth-order mixed integro-differential equations, Math. Probl. Eng. **2013**, 832074 (2013).
45. I. Abu Irwaq, M. Alquran, M. Ali, I. Jaradat, and M.S.M. Noorani, Attractive new fractional-integer power series method for solving singularly perturbed differential equations involving mixed fractional and integer derivatives, Results in Physics **20**, 103780 (2021).
46. M. Alquran, M. Ali, M. Alsukhour, and I. Jaradat, Promoted residual power series technique with

Laplace transform to solve some time-fractional problems arising in physics, Results in Physics **19**, 103667 (2020).

47. M. Ali, M. Alquran, and M. Mohammad, Solitonic solutions for homogeneous KdV systems by homotopy analysis method, Journal of Applied Mathematics **2012**, 569098 (2012).
48. H. Eltayeb and A. Kilicman, A note on double Laplace transform and telegraphic equations, Abst. Appl. Anal. **2013**, 932578 (2013).
49. L. Debnath, The double Laplace transforms and their properties with applications to Functional, Integral and Partial Differential Equations, Int. J. Appl. Comput. Math. **2**, 223-241 (2016).
50. H. Eltayeb and I. Bachar, A note on singular two-dimensional fractional coupled Burgers' equation and triple Laplace Adomian decomposition method, Boundary Value Problems **2020**, 129 (2020).
51. G. Adomian, A review of the decomposition method in applied mathematics, Journal of Mathematical Analysis and Applications **135**(2), 501-544 (1988).
52. S.A. Khuri, A Laplace decomposition algorithm applied to class of nonlinear differential equations, J. Math. Appl. **4**, 141-155 (2001).