Romanian Reports in Physics **74**, 403 (2022)

SUPPRESSION OF ULTRASHORT PULSE COLLAPSE BY QUANTUM FLUCTUATIONS IN BORN-INFELD MODELS ARISING FROM STRING THEORY AND HEISENBERG-EULER LAGRANGIAN

M.B. BELONENKO[a], N.N. KONOBEEVA[b]

Volgograd State University, University Avenue 100, 400062, Volgograd, Russia
E-mail[a]: *belonenko.mikhail@volsu.ru*
E-mail[b]: *yana_nn@volsu.ru*

Received January 10, 2022

Abstract. Based on the previously proposed model of accounting for quantum fluctuations for the Born-Infeld models, which arise from the string theories (strings and superstrings) and Heisenberg-Euler Lagrangian, the dynamics of an ultrashort optical pulse is considered. The Lagrangian contains the fourth degree in field strength. Higher derivatives are also taken into account in the Lagrangian. The dynamics of an ultrashort pulse is considered in the approximation of slowly varying amplitudes and phases. The terms arising due to the presence of higher derivatives in the Lagrangian stabilize the pulse and increase its lifetime until collapse.

Key words: string theories, quantum fluctuations, electromagnetic pulse, collapse.

1. INTRODUCTION

From the very beginning of the existence of electrodynamics in its modern form, there was a problem of the field of a point charge. Namely, in the Coulomb's law, in the case of point charge the field tends to infinity. To solve this problem, both nonlinear theories and theories in which higher derivatives are present in the Lagrangian have been proposed [1–3]. Note that nonlinear theories arise naturally, as theories for effective action in various models [4–7]. Recently, the interest of researchers is directed to the study of field theories with higher-order derivatives. The most widespread theory is the so-called Lee-Wick electrodynamics, which is Lorentz-invariant. Its complexity is demonstrated in recent works, for example in [8]. One of the most remarkable features of this electrodynamics is the fact that it leads to finite self-energy of a point charge in 3+1 dimensions [1, 4, 9]. It is worth mentioning other interesting results concerning the Lee-Wick theories [10–17]. It is shown in Ref. [18] that in the lowest order (one-loop correction), the standard Lee-Wick term appears as a quantum correction, quadratic in the gauge field in the proposed model, in the low-energy regime. We also note that terms with higher derivatives appear in the construction of the effective action in classical quantum electrodynamics [19]. In other words, we can say that one can not use theories of the Lee-Wick type, but turn to the corrections arising from ordinary quantum electrodynamics.

Another interesting approach is associated with the construction of effective Lagrangians arising from string and superstring theories [6, 7, 20]. Both terms leading to nonlinear equations, which are analogous to Born-Infeld electrodynamics [21–23], and terms with higher derivatives appear in a natural way. These terms appear when calculating the effective action in the low energy limit. Although the structure of such terms is different for the case of the Born-Infeld and Lee-Wick theories for some configurations of electromagnetic fields, they can be described within a single scheme.

It is also well known that the Maxwell's equations in vacuum, at high intensities of electric and magnetic fields, become nonlinear due to the exchange of virtual electron-positron pairs [24]. In the general case, this effect is described by the Heisenberg-Euler Lagrangian [25–26] from which it is easy to obtain an expression for the effective Lagrangian of the electromagnetic field. It should be noted that many studies have been carried out on the effects of higher orders. Usually, the research is carried out based on the approximation of slowly varying amplitudes and phases (SVAP). For example, in Ref. [27], the authors obtain the instability of light bullets within the framework of the nonlinear electrodynamics; light bullets either experience dispersive spreading (with the maximum amplitude tending to zero) or collapse in a finite time.

In this paper, we demonstrate the possibility of suppressing pulse collapse by introducing quantum corrections, which inevitably lead to terms with higher derivatives. Note that our consideration concerns only the possibility of suppressing the pulse collapse with allowance for higher derivatives in the effective Lagrangian. All the aforementioned theories lead to pulse collapse even when considering higher degrees of expansion in scalar and pseudoscalar invariants. That is why the question arise of taking into account the higher derivatives in action. Since the pulse is considered to be of high-intensity, *i.e.* contains a certain number of photons, which is much more than one, then we limited ourselves to the classical limit. Quantum effects are not considered precisely because of the considerations mentioned above. The study starts with quantum Lagrangians, which were introduced before us, and the work investigates the classical propagation of a high-intensity short (from 10 to 104 field oscillations) pulse.

The problem considered in this work is rather mathematical and abstract in nature, in particular, there is no comparison with experimental data. We note that the possibility of observing the effects of nonlinear electrodynamics in vacuum has been discussed for a long time; see, for example [28–31].

2. STATEMENT OF THE PROBLEM AND BASIC EQUATIONS

The Lagrangian containing the gauge field A_μ and a single massive charged fermion field ψ, in the case of the Lee-Wick theories, has the form:

$$L = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}\left[i\partial - M - \gamma^\mu g\left(A_\mu - \frac{1}{m_p^2}\partial^\nu F_{\mu\nu} \right) \right]\psi\,, \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength, M is the fermion mass, g is the electric charge, and m_p is the mass parameter that is responsible for the relationship between the fermion field and the photon field in the high-energy regime. Summation is implied over repeated indices. The metric is selected flat with a signature (+---). We consider that $h / 2\pi = 1$.

The full effective action $S_{eff}[A]$ can be written as [18]:

$$\begin{aligned} S_{eff}\left[A\right] = \int \mathrm{d}^4x \Bigg[&-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{g^2C}{4}F^{\mu\nu}F_{\mu\nu} - \frac{g^2C}{m_p^2}F_{\mu\nu}\partial^\alpha\partial_\alpha F^{\mu\nu} + \\ &+\frac{g^2C}{2m_p^4}\partial_\mu\partial^\beta F_{\nu\beta}\partial^\mu\partial_\alpha F^{\nu\alpha} + L_{F^4}\ldots \Bigg], \end{aligned} \tag{2}$$

where L_{F^4} contains the term of the fourth order in the field tensor, and the constant used in the renormalization is defined as:

$$C = \frac{1}{12\pi^2}\int_{\varepsilon>0}^{\infty} \mathrm{d}\tau\, \tau^{-1}e^{-M^2\tau} \tag{3}$$

In terms of normalized quantities, the action (2) has the form:

$$S_{eff}\left[A\right] = \int \mathrm{d}^4x \left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{g^2}{m_p^2}F_{\mu\nu}\partial^\alpha\partial_\alpha F^{\mu\nu} + L_{F^4}\ldots \right] \tag{4}$$

The second term on the right-hand side of Eq. (4) includes higher-order derivatives of the photon field. It is obtained from the quantum corrections of the model (1). This is the so-called member of Lee-Wick theory. The third term in Eq. (4) describes the lowest-order nonlinear corrections to Maxwell's theory, which in the case of quantum electrodynamics are known as the Euler-Heisenberg term [26, 32]. In the limit $m_p \to \infty$ it can be written in the following form:

$$L_{F^4} = \alpha\left[\left(\boldsymbol{E}^2 - \boldsymbol{B}^2 \right)^2 + 7\left(\boldsymbol{E}\cdot\boldsymbol{B} \right)^2 \right], \quad \alpha = \frac{g^4}{180\pi^2M^4}\,. \tag{5}$$

E and ***B*** in this case are the classical vectors of the electric field strength and magnetic induction, respectively.

The effective Lagrangians following from string theories [6] have the following form for superstrings and bosonic strings, respectively:

$$L=\sqrt{-\det\left(\delta_m^n+F_m^n\right)}-\frac{1}{96}\left(\partial^a\partial^b F_m^n\partial_a\partial_b F_n^l F_l^r F_r^m+\frac{1}{2}\partial^a\partial^b F_m^n F_n^l\partial_a\partial_b F_l^r F_r^m-\right.$$
$$\left.-\frac{1}{4}\partial^a\partial^b F_m^n F_m^n\partial_a\partial_b F_l^r F_l^r-\frac{1}{8}\partial^a\partial^b F_m^n\partial_a\partial_b F_m^n F_l^r F_l^r\right)+\text{higher derivatives} \quad (6)$$

$$L=\sqrt{-\det\left(\eta_m^n+F_m^n\right)}-\frac{1}{48}\left(F_k^l F_k^l\partial^a F_m^n\partial_a F_m^n+8F_k^l F_l^m\partial^a F_m^n\partial_a F_n^k-\right.$$
$$\left.-4F_l^a F_l^b\partial^a F_m^n\partial_b F_m^n\right)+\text{higher derivatives} \quad (7)$$

Note that the first term associated with the determinant coincides up to the coupling constant when expanding in a series with the first term in Eq. (5) at ***EB*** = 0.

The system of Maxwell's equations in the absence of free charges and currents can be written (hereinafter $c = 1$):

$$\Box\boldsymbol{E}=-\mu_0\left(\frac{\partial^2\boldsymbol{P}}{\partial t^2}+\nabla\left(\overline{\nabla}\cdot\boldsymbol{P}\right)+\frac{\partial}{\partial t}\left(\overline{\nabla}\times\boldsymbol{M}\right)\right),$$
$$\Box\boldsymbol{B}=-\mu_0\left(\overline{\nabla}\times\left(\overline{\nabla}\times\boldsymbol{M}\right)+\frac{\partial}{\partial t}\left(\overline{\nabla}\times\boldsymbol{P}\right)\right) \quad (8)$$

We take into account that $\boldsymbol{P}=\delta L/\delta\boldsymbol{E},\ \boldsymbol{M}=-\delta L/\delta\boldsymbol{B}$, where as L, as noted above, we can take the classical Lagrangian. It is easy to obtain the expressions for the polarization ***P*** and magnetization ***M*** in the case of the Lee-Wick model:

$$\boldsymbol{P}=\alpha\left(2\left(\boldsymbol{E}^2-\boldsymbol{B}^2\right)\boldsymbol{E}+7\left(\boldsymbol{E}\cdot\boldsymbol{B}\right)\boldsymbol{B}\right)-\beta\Box\boldsymbol{E},$$
$$\boldsymbol{M}=\alpha\left(-2\left(\boldsymbol{E}^2-\boldsymbol{B}^2\right)\boldsymbol{B}+7\left(\boldsymbol{E}\cdot\boldsymbol{B}\right)\boldsymbol{E}\right)+\beta\Box\boldsymbol{B}, \quad (9)$$
$$\beta=\frac{g^2}{m_p^2}.$$

The corresponding formulas for the string models have a similar form, but include terms with derivatives of the form $\boldsymbol{E}\boldsymbol{E}^2\boldsymbol{E}$ and $\boldsymbol{E}\boldsymbol{E}_x\boldsymbol{E}_x$. The further form of the equations can be greatly simplified by choosing a special type of solutions for electromagnetic waves. Let the electric field have only one nonzero component:

$$\boldsymbol{A}=\left(0,A\cdot e^{i\theta},0\right),A=A\left(x,y,z,t\right),\quad\theta=\omega t-kz, \quad (10)$$

$$\boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t},\ \boldsymbol{B} = \mathrm{rot}\,\boldsymbol{A},\qquad \boldsymbol{E}\cdot\boldsymbol{B} = 0. \tag{11}$$

We use the following approximation:

$$\left|\frac{\partial A}{\partial t}\right| \ll \omega, \left|\frac{\partial A}{\partial \chi}\right| \ll k, \chi = \left(x, y, z\right). \tag{12}$$

Formulas (10) and (12) specify the approximations used in this work.

Thus, the effective equation with the allowance for quantum corrections in the approximation of slowly varying amplitudes and phases has the form:

$$\gamma A + iA_\tau + \frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}\left(\rho\frac{\mathrm{d}A}{\mathrm{d}\rho}\right) - \delta_1 A_{tt} - i\delta_2 A_{ttt} + i\delta_3\left|A\right|^2\cdot A_t + \gamma_1\left|A\right|^2\cdot A + i\delta_4 A^2\cdot A_t^* = 0 \tag{13}$$

$$\begin{gathered}\gamma = 2\omega^2 - 2k^2 + 4\beta\omega^2\left(\omega^2 + k^2\right) - 4\beta k^2\left(\omega^2 + k^2\right)\\ \delta_1 = 2 + 8\beta\omega^2 - 4\beta k^2, \delta_2 = -4\omega\beta,\\ \delta_3 = \left(-72\omega k^2 + 48\omega^3\right)\cdot\alpha, \delta_4 = -12\alpha\omega\left(k^4 + \omega^4\right),\\ \gamma_1 = 12\alpha\left(k^4 + \omega^4\right),\\ \tau = -\frac{\partial l}{\partial \omega}\cdot t - \frac{\partial l}{\partial k}\cdot z, l = -4\omega k - 8\omega^2 k^2\beta,\\ \rho = 2x^2 + 2y^2\cdot\left(4\beta\cdot\left(k^2 - \omega^2\right)\right)^{0.5}.\end{gathered} \tag{14}$$

Here we first change the scale of the variables x and y, and then we make the transition to a cylindrical coordinate system, and also neglect the derivative with respect to the angle, since the cylindrical symmetry in the field distribution is preserved (similarly was done in [33]).

The derivation of Eq. (13) from the equations obtained above has been repeatedly described in the literature; for example, in [34–39]. Also in [40], one can find the effective Lagrangian, which corresponds to (13) at $\delta_2 = \delta_4 = 0$. Note that for the models following from the string theories $\delta_2 = 0$.

3. MAIN RESULTS OF NUMERICAL SIMULATION AND DISCUSSION

Equation (13) is solved numerically by the Dufort method [41], which is an explicit scheme. The initial conditions on the first layer are set in the form:

$$A_y\Big|_{\tau=0} = Q\cdot\sin\left(t\right)\exp\left(-\frac{t^2}{\gamma^2}\right)\exp\left(-\frac{\rho^2}{\gamma_\rho^2}\right). \tag{15a}$$

The initial condition at the second time level has the form:

$$A_y\big|_{\tau=\text{ht}} = Q\cdot\sin\left(t+ht\cdot v\right)\exp\left(-\frac{\left(t-ht\cdot v\right)^2}{\gamma^2}\right)\exp\left(-\frac{\rho^2}{\gamma_\rho^2}\right) \tag{15b}$$

Here Q is the amplitude, v is the initial velocity of the pulse, ht is the time step, γ is the pulse width along the propagation direction, and γ_ρ is the pulse width in the direction perpendicular to the propagation direction.

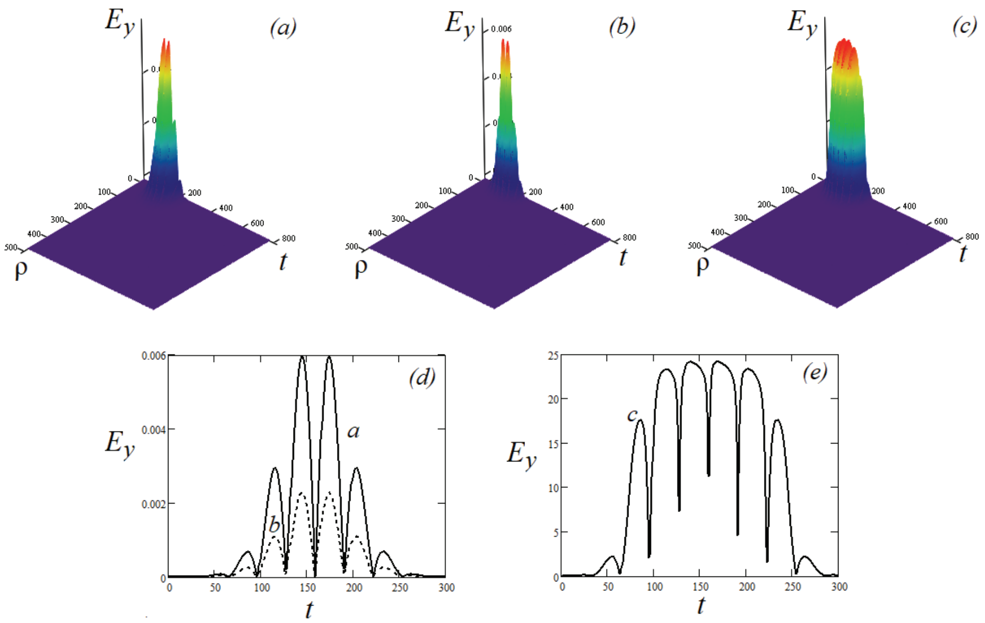

Fig. 1 – The evolution (*a–c*) and slices with non-zero linear and non-linear variance (δ_2 and δ_3, $\delta_4 \neq 0$): a) $\tau = 0$ r.u.; b) $\tau = 600$ r.u.; c) $\tau = 1200$ r.u. Figures (*d–e*) show slices: d) at $\tau = 0$ r.u. and $\tau = 600$ r.u. (curves *a* and *b*, respectively); *e*) $\tau = 1200$ r.u.

As can be seen from Fig. 2, taking into account the linear dispersion leads to pulse stabilization, more precisely, to suppression of the collapse (Fig. 2b) by the quantum fluctuations. Calculation is made in the wide range of parameters.

It should be noted here that the calculations, the typical results of which are shown in Fig. 2, are carried out in two different ways. In one case, the coefficients δ_3, δ_4 are changed independently by setting different k and ω. In another case, the quantities k and ω are considered related to each other, and ω is found after specifying k from the dispersion equation of linearized Eqs. (6). In both cases, at zero value of the parameter β, which arises as a result of taking into account the quantum corrections in action (1), a collapse of the pulse is observed.

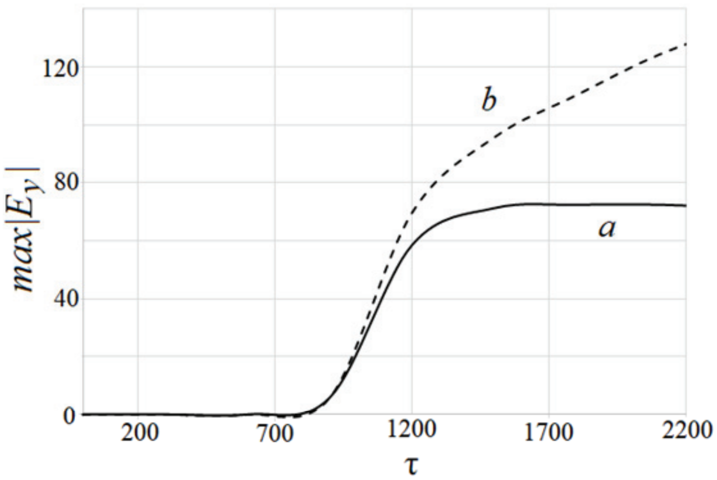

Fig. 2 – Dependence of the maximum of $|E|$ on time τ for different values of dispersion coefficients: a) $\delta_2 \neq 0$, δ_3, $\delta_4 \neq 0$; b) $\delta_2 = 0$, δ_3, $\delta_4 \neq 0$.

This is consistent with both the previously obtained results [27, 38–42] and our results, which are obtained without the approximation of slowly varying amplitudes and phases (9) [43]. This is also consistent with other results, taking into account the nonlinearities of a different type, also obtained outside the SVAP approximation [44–45]. Thus, the pulse undergoes a collapse without taking into account the effect of the linear dispersion. Taking into account the linear dispersion leads to stabilization of the collapse and to the termination of the growth of the pulse amplitude. A similar dispersion is introduced earlier [45] and can be interpreted as an analogue of the Heisenberg-Euler Lagrangian in rapidly changing fields. Note that the approach we have proposed extends to this case, only it is necessary to take into account another form of the dependence of the coefficient δ_2 on ω. All of the above allows us to conclude that the quantum corrections to the Heisenberg-Euler action stabilize the pulse and prevent the collapse.

It is necessary to pay special attention to the fact that in models following from the string theories, the term with δ_2 is exactly equal to zero. Thus, the results indicate that, within the framework of the mentioned approximations and of the effective Lagrangians of string theories, the ultrashort pulse is unstable. Obviously, this problem can be solved in two ways: by the introduction of the correction terms for rapidly varying fields [19], or the inclusion of additional terms in the expansion of the effective Lagrangian in powers of the strength tensor $F_{\mu\nu}$ and its derivatives. If we consider the pulse dynamics against the background of a random electromagnetic field in the framework of string theories, *i.e.* $\langle \boldsymbol{E} \rangle = 0$, but $\langle \boldsymbol{EE} \rangle \neq 0$, then the term with δ_2 appears automatically and the problem can be solved within the framework of the approximations made.

At the same time, a wide range of issues remained uncovered in the work. Still remaining within the framework of the action following from the string theory with the quantum corrections, and taking into account that collapse does not occur even in the lowest order in quantum corrections, it is necessary to take into account higher order corrections, namely, the diffraction in the transverse direction and the nonlinear diffraction (*i.e.*, terms that depend on the spatial derivatives of the pulse amplitude and on the pulse amplitude itself). *A priori*, the contribution of these effects is unclear and can lead both to the pulse collapse and to its decay. It is also important to take into account the time derivatives of nonlinear terms and of a higher order.

We note that the question of the particles production in the pulse field during its evolution remained outside the scope of our consideration. This effect, namely the Schwinger effect, is important at the stage of the pulse collapse and leads to nonlinear damping effects, which can additionally stabilize the considered pulse. When considering this effect, it is also be important what type of particle production is considered [46–47].

Acknowledgments. The authors thank the Ministry of Science and Higher Education of the Russian Federation for the numerical modeling and parallel computing support under the government task (no. 0633-2020-0003).

REFERENCES

1. A. Accioly and M. Dias, Phys. Rev. D **70**, 107705 (2004).
2. A. Accioly, P. Gaete, J. Helayel-Neto, E. Scatena, and R. Turcati, Mod. Phys. Lett. A **26**, 1985 (2011).
3. R.R. Cuzinatto, C.A.M. de Melo, and P.J. Pompeia, Annals of Phys. **322**, 1211 (2007).
4. J. Frenkel, Phys. Rev. E **54**(5), 5859 (1996).
5. B. Grinstein, D. O'Connell, and M.B. Wise, Phys. Rev. D **77**, 025012 (2008).
6. A.A. Tseytlin, *Born-Infeld action, supersymmetry and string theory. In The Many Faces of the Superworld*, Shifman, M. Ed.; World Scientific Publishing Company, Singapore, 2000.
7. A.A. Tseytlin. Nuclear Phys. **B276**, 391 (1986).
8. D. Anselmi and M. Piva, J. High Energy Phys. **06**, 066 (2017).
9. F.A. Barone, G. Flores-Hidalgo, and A.A. Nogueira, Phys. Rev. D **91**, 027701 (2015).
10. C.A. Bonin, R. Bufalo, B.M. Pimentel, and G.E.R. Zambrano, Phys. Rev. D **81**, 025003 (2010).
11. R. Baginski and B. Santos, Mod. Phys. Lett. A **26**, 1909 (2011).
12. F.A. Barone and A.A. Nogueira, Int. J. Mod. Phys.: Conf. Ser. **41**, 1660134 (2016).
13. C. Bambi, L. Modesto, and Y. Wang, Phys. Lett. B **764**, 306 (2017).
14. L. Modesto, Nucl. Phys. B **909**, 584 (2016).
15. L. Modesto and I.L. Shapiro, Phys. Lett. B **755**, 279 (2016).
16. F. Wu and M. Zhong, Phys. Rev. D **78**, 085010 (2008).
17. M. Dias, A.Yu. Petrov, C.R. Senise Jr., and A.J. da Silva, Int. J. Mod. Phys. A **30**, 1550113 (2015).
18. L.H.C. Borgesa, F.A. Barone, C.A.M. deMelo, and F.E. Barone, Nuclear Physics B **944**, 114634 (2019).
19. S.G. Mamaev, V.M. Mostepanenko, and M.I. Eides, Sov. J. Nucl. Phys. **33**, 569 (1981).

20. S.V. Ketov, *Many faces of Born-Infeld theory*, arXiv:hep-th/0108189v1 (2001).
21. S. Cecotti and S. Ferrara, Phys. Lett. B **187**, 335 (1987).
22. R.R. Metsaev, M.A. Rakhmanov, and A.A. Tseytlin, Phys. Lett. B **193**, 207 (1987).
23. M.B. Belonenko, N.N. Konobeeva, and A.V. Zhukov, Mod. Phys. Lett. A **35** (27), 2050221 (2020).
24. L.D. Landau and E.M. Lifshitz, *Quantum electrodynamics*, 2nd ed., Pergamon, Oxford, 1989.
25. W. Heisenberg and H. Euler, *Folgerungen aus der Diracschen theorie des positrons*, Z. Phys. **98**, 714 (1936).
26. J. Schwinger, Phys. Rev. **82**, 664 (1951).
27. M. Desaix, D. Anderson, and M. Lisak, J. Opt. Soc. Am. B **8**, 2082–2086 (1991).
28. V.I. Beskin, A.V. Gurevich, and Ya.N. Istomin, *Physics of the pulsar magnetosphere*, Cambridge University Press, Cambridge, 1993.
29. M.F. Curtis, Rev. Mod. Phys. **54**, 1 (1982).
30. A.K. Harding, Science **251**, 1033 (1991).
31. C. Kouveliotou, S. Dieters, T. Strohmayer, J. van Paradijs, G.J. Fishman, C.A. Meegan, K. Hurley, J. Kommers, I. Smith, D. Frail, and T. Murakami, Nature **393**, 235 (1998).
32. G.V. Dunne, *Heisenberg-Euler effective Lagrangians: basics and extensions*, arXiv:hep-th/0406216 (2004).
33. A.V. Zhukov, R. Bouffanais, E.G. Fedorov, and M.B. Belonenko, J. Appl. Phys. **114**, 143106 (2013).
34. A. Hasegawa, *Introduction to plasma instabilities*, in: *Plasma instabilities and nonlinear effects. Physics and chemistry in space*, Springer, Berlin, Heidelberg, 1975.
35. Y.S. Kivshar and G.P. Agrawal, *Optical Solitons*, Academic Press, San Diego, 2003.
36. N.N. Rozanov, JETP **76**, 991 (1993).
37. G. Brodin, M. Marklund, and N. Stenflo, Phys. Rev. Lett. **87**, 171801 (2001).
38. G. Brodin, L. Stenflo, D. Anderson, M. Lisak, M. Marklund, and P. Johannisson, Phys. Lett. A **306**, 206 (2003).
39. P.K. Shukla, B. Eliasson, and M. Marklund, Opt. Comm. **235**, 373 (2004).
40. M. Marklund and P.K. Shukla, Rev. Mod. Phys. **78**, 591 (2006).
41. L. Wu, SIAM J. Numer. Anal. **33** (4), 1526 (1996).
42. M. Marklund, P.K. Shukla, and B. Eliasson, *The intense radiation gas*, Europhys. Lett. **70**, 327 (2005).
43. M.B. Belonenko and N.N. Konobeeva, *Instability of localized pulses in nonlinear electrodynamics*, arXiv:1901.10126 (2019).
44. M.B. Belonenko and N.N. Konobeeva, *Collapse of ultra-short pulse of electromagnetic field within non-linear electrodynamics*, arXiv:1909.05676 (2019).
45. M.B. Belonenko and N.N. Konobeeva, *Extremely short electromagnetic field pulse in supersymmetric electrodynamics*, arXiv:2008.02079 (2020).
46. C. Bachas and M. Porrati, Phys. Lett. B **296** (1–2), 77 (1992).
47. B. Zwiebach, *A first course in string theory*, Cambridge University Press, Cambridge, 2009.