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STUDY OF THE FRICTION TORQUE IN THE BEARING OF A FLYWHEEL IN THE GENERAL PHYSICS LABORATORY

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Abstract. In one of the standard laboratory exercises a system of two bodies is studied – a flywheel and a weight connected to its axis by means of a cord. Our aim is to verify that the moment of the frictional forces in the flywheel bearing is constant. In order to achieve this goal, we study the movement of the weight and investigate it through video recording. Through a statistical test, the coefficient of determination R^2, we prove that the acceleration of the weight is constant and, hence, the flywheel also rotates with constant acceleration which, on its side, means that the moment of friction is constant.

Key words: uniformly accelerated motion, friction, bearing, coefficient of determination R^2.

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1. INTRODUCTION

General physics is one of the main courses for engineering students [1, 2]. The discipline itself, by definition, gives young researchers a broad view of the application of various natural phenomena and laws in technology and industry. On the one hand, this goes hand in hand with mathematics and precise theoretical descriptions, but on the other hand, emphasis should be placed on the students' practical skills in verifying the various laws. These skills are learned and developed through the laboratory practicum of the physics course. The rapid development of science and technology, however, necessitates the periodic updating of this practicum. Some exercises should be dropped, others should be renewed, and new ones should be introduced. Here, the new capabilities of technology must be used for the measurement of various quantities and for the analysis of the results. Often, the automation required for this involves rather expensive resources, such as computer interface, and is unaffordable for one department, especially when it has to be implemented for several new exercises.

Then comes the idea of using much more accessible means available to each of us. These are our cell phones [3–6]. Every student has a phone with a camera, they can take pictures, shoot videos and upload them to a computer. There are programs available to view these time-stepped videos. This allows for much larger data sets to be available and analyzed using affordable statistical programs. This gives us the opportunity to study various dependencies in more detail to calculate their magnitudes, and to evaluate their statistical significance. This means that we can take an idea from an old lab exercise, develop it in a new way, and renew that exercise using affordable but reliable technical means. In the Department of Applied Physics of the Technical University of Sofia, we have already made such updates more than once [7, 8].

Another exercise from our general physics laboratory is considered in the present work. Here, we focus on friction forces in bearings. In the classic exercise, a system of two bodies is considered: a flywheel and a weight connected to it by a cord. The flywheel is fixed at a certain height so that it can rotate freely with the help of a bearing. The weight descends and sets the flywheel in motion through the cord. The time for the weight to reach the floor and the time for the flywheel to stop after that is measured. From there, using the measured times and also the values of other quantities [9, 10] the moment of inertia of the flywheel and the moment of the frictional force in the bearing can be calculated.

In this experiment it is assumed that the frictional force in the bearing is constant with time, and hence the downward motion of the weight is uniformly accelerated. This is not checked in the classical exercise and there is no way to do it with the available equipment. Our idea is to use a cellphone to record the descent of the weight and, through statistical analysis of the data, to check whether the acceleration is constant and hence whether the frictional force in the flywheel bearing is constant over time. This becomes a new and easy to measure and implement exercise, using the old experimental setup, but with a new way of acquisition and analysis of data and a new general idea. It also helps that the flywheel is sufficiently inertial and the descent of the weight is very slow. No need to use a high-speed camera. An ordinary phone does the job. Here the flywheel is used only as an aid. In motion analysis, we consider only the weight and use the relationship of the constant frictional torque in the bearing with the constant downward acceleration of the weight.

In addition to this Introduction, the present work contains: a rationale for the study – evidence of the relationship of the constant moment of frictional force in the bearing to the constant acceleration of the weight, Section 2; description of the experiment, Section 3; weight motion model, Section 4; statistical processing of experimental data, Section 5; and Conclusion.

2. THEORETICAL JUSTIFICATION OF THE STUDY

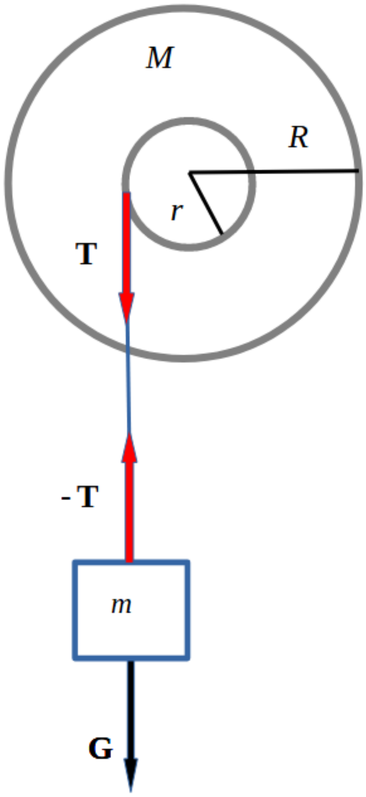

Fig. 1 – Scheme of the experimental setup.

In this Section we present a short theoretical consideration of the physical processes in regard*. In Fig. 1 are displayed the flywheel and the weight, attached to it by a cord. This cord we regard as very thin and massless. So, there are two equations of motion, one for the flywheel and one for the weight, which describe the motion of the system. Let us assume that the positive direction of the flywheel's axis is directed to the reader, and the positive direction of the vertical axis regarded to the motion of the weight is directed downwards. In this manner, the two positive directions are consistent in respect to the right-hand rule. The two equations of motion read as follows:

$$\vec{\tau} + \vec{r} \times \vec{T} = I\vec{\alpha} \;, \tag{1}$$

*For theoretical analysis of similar physical systems we refer the reader to [11, 12].

and

$$-\vec{T}+\vec{G}=m\vec{a}\ , \tag{2}$$

where the equation (1) describes the rotation of the flywheel, and the equation (2) corresponds to the motion of the weight. In the first equation $\vec{\tau}$ denotes the friction torque in the bearing, $\vec{T}$ is the tension of the cord, $\vec{r}$ is the radius-vector from the flywheel axis to the cord, I is the moment of inertia of the flywheel, and $\vec{\alpha}$ is the angular acceleration. In the second equation $\vec{G}=m\vec{g}$ is the gravity force acting on the weight, m is the mass of the weight, and $\vec{a}$ is its linear acceleration.

Our goal is to study the friction torque $\vec{\tau}$ in regard to the motion of the whole mechanical system. As $\vec{\tau}$ is directed to the negative direction of the flywheel's axis during all the time of motion, then we need to obtain an equation, which presents τ as a function of the linear acceleration a (which is measured), and the other parameters of this system: the moment of inertia I, the mass m, the radius r, and gravity acceleration g, which are constants, accordingly. To achieve this goal we project the above equations on the flywheel's axis and vertical axis, respectively, keep in mind the chosen positive directions. Then we get the projected equations as follows:

$$-\tau+rT=I\alpha\ , \tag{3}$$

and

$$-T+mg=ma\ . \tag{4}$$

After that one can express the tension from equation (4) and it reads: $T=m(g-a)$. The latter formula we replace in equation (3) and also we make use of the relation $\alpha=a/r$. Finally, we obtain the wanted equation for the friction torque:

$$\tau=rm(g-a)-I\frac{a}{r}\ . \tag{5}$$

The main conclusion that we could make is that if the linear acceleration a is a constant during the motion, then the friction torque τ in the bearing is a constant, too.

3. EXPERIMENT

As it was mentioned above that the experimental set-up consists of two parts: an inertial flywheel and a weight that sets the flywheel in motion by means of a cord, Fig. 1. The laboratory instruction manual [9, 10] describes the motion of the system and derives the main dependencies using an energetic approach. The idea is to realize two motions. First, uniformly accelerated motion without initial speed of the weight, which descends from a certain height downwards and at the end of the first motion falls on the floor. During this stage of the motion, through the cord, the weight sets the flywheel in a uniformly accelerated rotational motion. Second, uniformly accelerated rotation of the flywheel until it stops under the action of the friction torque in the bearing. From the two motions, expressions can be derived for the moment of inertia of the flywheel and for the moment of frictional forces in the bearing.

As already stated above, in the derivations it is assumed that the moment of the frictional force is constant with time, which means that the acceleration of the system is also constant. Accordingly, the downward motion of the weight is with constant acceleration. This is a good assumption and makes the task model easier, but we decided to test it. We concentrate only on the first motion – the uniformly accelerated descent of the weight.

Due to its connection by the cord to the flywheel and due to its great inertia, the acceleration of the weight is very small, and since the motion starts from rest, the values of its speed are also small. This led us to the idea that the descent of the weight could be recorded with the camera of a simple mobile phone as a video and the necessary data could be downloaded from it.

Filming is done as follows. A linear metric scale is placed just behind the cord with the weight, Fig. 2, so that when the video is taken, it shows the position of the descending weight at any given moment. The data is taken from the video, namely the position of the weight at different time values. This is done over a certain time step. This method of operation allows the capture of a larger set of data, and, hence, statistical processing.

To analyze the resulting video file we use OpenShot Video Editor[†]. It is free and open-source, available for download on Linux, OS X, and Windows. First, we launch OpenShot Video Editor. From File/Import Files we load the video file in Project files. With the right mouse button, we click on the selected file and select Add to Timeline, Fig. 3. The video file is loaded in the Timeline and in the Video Preview, where we can zoom in or out using the scroll wheel of the mouse. The video is started and stopped with the Space bar, and with the left and right arrows steps of 0.01 seconds can be made.

4. MODEL

As a consequence of our main hypothesis, that the friction torque in the bearing is constant, the motion of the weight is uniformly accelerated. It also starts with zero velocity. Hence, the path h travelled in vertical direction by the weight is a quadratic function of time

$$h = \frac{1}{2}at^2. \tag{6}$$

[†] https://www.openshot.org.

Fig. 2 – Photo of the experimental setup.

If we take the square root of both sides we obtain the following linear model

$$y \equiv \sqrt{h} = \sqrt{\frac{1}{2}a}t \equiv bx, \tag{7}$$

which has one parameter $b = \sqrt{(1/2)a}$.

5. REDUCTION OF DATA

Our aim is to apply the chosen model (Sec. 4) to fit the experimental data, to obtain an estimate for the value of the parameter of the model b, and, hence, to estimate the acceleration a, and, at the end, to access the quality of the model. If the chosen model for the law of motion of the weight is adequate that would support the hypothesis that the acceleration is constant and hence the torque of the friction forces in the bearing of the flywheel is also constant.

The statistical analysis of the experimental data that we make here follows the steps given in [7]. To model the experimental data obtained from the video we apply the simple regression package implemented in Excel. For details on the production

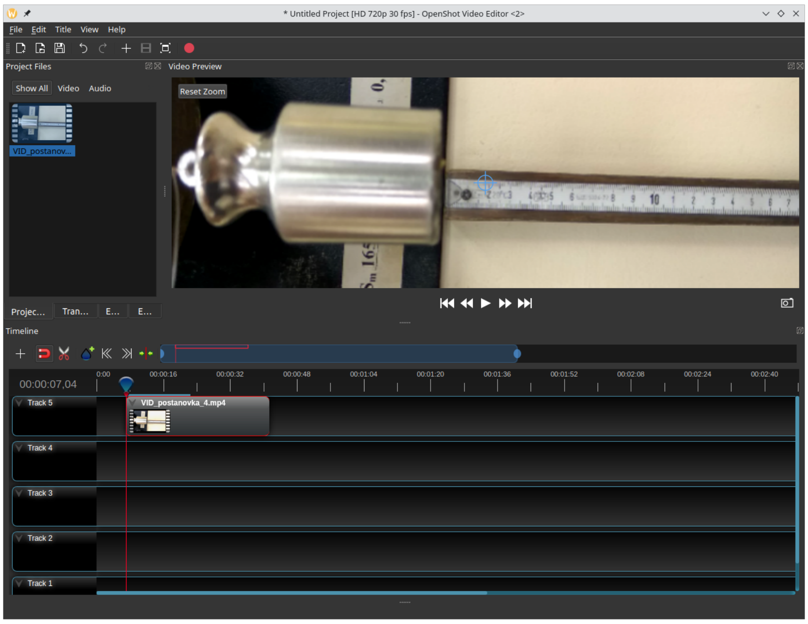

Fig. 3 – Interface of OpenShot Video Editor.

of the plots, we can refer the reader to the following video lecture [13]. In this same lecture, and also in [14–16], detailed descriptions of the interpretation of the results obtained by the Regression function of Excel can also be found[‡].

On the role of the standardized residuals for the identification of outlier, on the application of the F-test for the estimation of the overall significance of the fit, and in general on simple regression analysis please refer to the following books [21–23] and video lectures [13, 15].

The results from the regression conducted in Excel are given on Fig. 4. One of the major criteria used to evaluate the quality of the approximation, *i.e.* the extent to which the selected model can explain the obtained experimental data is the coefficient of determination R^2. Its value is $R^2 = 0.999$, circled in blue in Fig. 4. It means that the variance of 99.9% of the experimental points can be explained by the model which, we can say, is a pretty good result. As an additional estimate of the overall goodness of the fit we can consider the “Significance F”. Its value, circled in blue in Fig. 4, is practically zero up to 88 decimal places. This assessment also does not give us reason to question the applied model.

[‡]Excel can be very useful for data reduction [17–20]. Besides, its broad implementation makes it easily accessible.

SUMMARY OUTPUT	
Regression Statistics	
Multiple R	0.99987785
R Square	0.99975571
Adjusted R Square	0.97934755
Standard Error	0.11275179
Observations	50

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	2549.377065	2549.37706	200533.6258	1.4253E-88
Residual	49	0.622935309	0.01271297		
Total	50	2550			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
0	0.3327	0.0007	447.80981	3.61656E-90

Fig. 4 – Results of the “Regression” procedure.

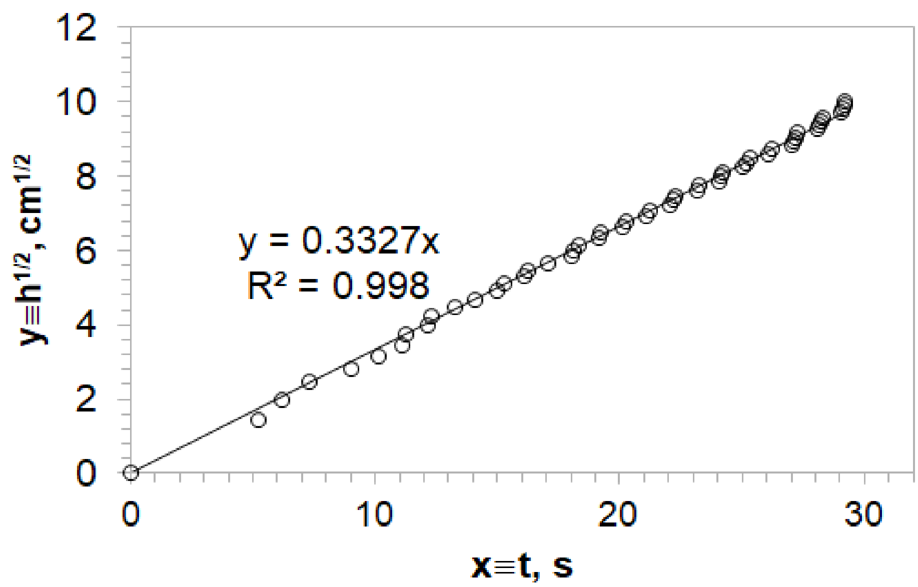

Fig. 5 – The experimental points and the model line.

Figure 5 shows the experimental points and the model line. Visually we can establish a good match, *i.e.* most of the experimental points lie close to the model

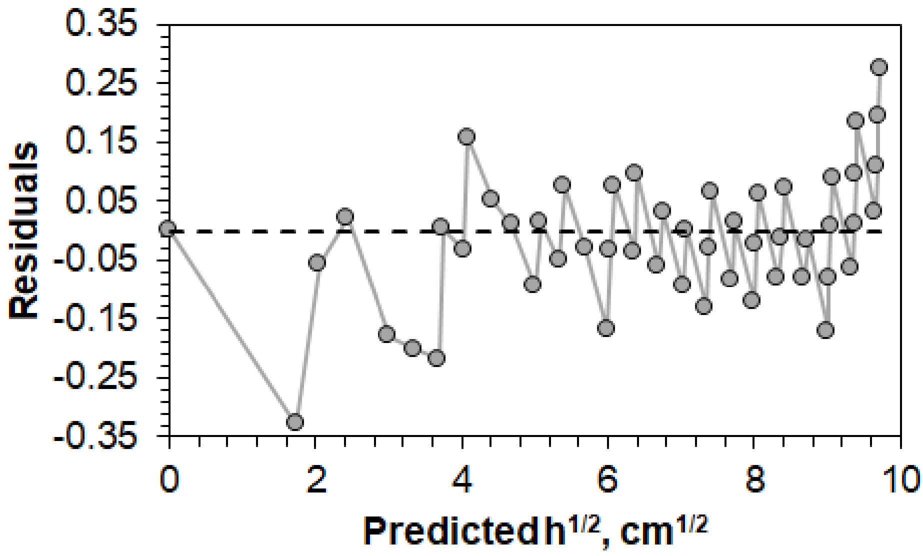

Fig. 6 – The residuals plus a reference line.

line.

The residuals from the approximation are shown in Fig. 6. In this plot we have included also as a reference a horizontal line through the zero. If the residuals show some pattern or non-constant variance (heteroscedasticity), it would be an indication that they are not independent. The latter would question the adequacy of the chosen model. If we neglect the second point as an obvious outlier we could say that the residuals are randomly distributed around the zero value. For more details on residual analysis we refer the reader to [14].

The second and the last point are apparent outliers. Indeed, their standardized residuals are -2.98 and 2.48, respectively, and, as usually accepted, values greater than 2-3 imply that the corresponding points are outliers.

6. CONCLUSION

In the present paper, we propose a modification of one of the standard laboratory exercises for the university course in general physics. In the laboratory exercise, a system of two bodies is considered – a flywheel and a weight connected to it by means of a cord. The standard laboratory exercise is aimed at investigating the rotational motion of the flywheel and related quantities such as moment of inertia and work done by the moment of frictional forces. The study we present is a verification of one of the main assumptions made in the theoretical description of the system's motion in this laboratory exercise, namely that the moment of the frictional forces in the flywheel bearing is constant.

We achieve this in the following way. We focus on the movement of the weight and investigate it through video recording. We model the position of the weight as a function of time using a quadratic law. Then we perform a statistical test that confirms that the experimental data are well modelled using a quadratic time law of motion. The conclusion that can be made is that the acceleration of the weight is constant, and it follows that the flywheel also rotates with a constant acceleration. In other words, we were able to prove experimentally that the moment of friction is constant.

Our proposed research can be considered, on the one hand, as a supplement to the standard laboratory exercise, and on the other hand, as an independent laboratory exercise that is focused on the kinematics of constant acceleration motion. In the second case, the flywheel acts as a retarder, which allows us to study the movement of the weight without using a high-speed camera.

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