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EXPERIMENTAL DETERMINATION OF THE INDUCTANCE OF A SOLENOID: AN EDUCATIONAL APPROACH

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Abstract. The proposed physics experiment is designed as an educational tool to enhance the understanding of fundamental principles in electromagnetism by enabling students to experimentally determine the inductance of a solenoid through electromagnetic induction without relying on moving conductors or magnets. The primary objective is to integrate theoretical knowledge about the relationships among important electromagnetic quantities with hands-on experimental methodologies for their measurement and evaluation. Rather than directly measuring the solenoid's inductance, the selected approach indirectly determines it by analyzing variations in magnetic induction as a function of electric current. The experimental data were thoroughly analyzed using Weighted Least Squares, regression, thus highlighting their pedagogical value and effectiveness in handling experimental uncertainties. Additionally, the experimental outcomes are validated through independent direct measurements, reinforcing both the reliability of the method and its educational significance.

Key words: inductance, magnetic induction, linear regression.

1. INTRODUCTION

The fundamental quantities in electromagnetism describe the relationship between the electric and magnetic fields, relate the characteristics of these fields to their sources, and explain their effects on electric charges and currents. Understanding electromagnetic induction and resulting electromotive forces is essential in physics education, electrical engineering, and numerous technological applications. Faraday’s law, one of the cornerstone principles of electromagnetism, describes how a change in magnetic flux through a closed loop induces an electromotive force (EMF) in the loop. Specifically, Faraday’s law states that the induced EMF is directly proportional to the rate at which magnetic flux changes, thus underpinning the operation of critical electrical devices such as inductors, transformers, electric generators, and various sensing technologies [1-3].

In educational settings, effectively illustrating Faraday’s law and the concept of inductance can significantly enhance students' comprehension of complex theoretical concepts. This study proposes a practical, accessible, and pedagogically meaningful experiment designed to introduce students to the fundamental principles of electromagnetic induction, focusing specifically on determining the inductance of a solenoid without relying on moving conductors or magnets. The experimental setup consists primarily of a solenoid connected to an adjustable DC power supply that allows precise control over the electric current flowing through the solenoid. The use of DC currents and the absence of moving conductor and magnets is a qualitative distinction compared to other commonly proposed experiments with LC-circuit [4]. Magnetic induction inside the solenoid is measured using a highly sensitive, three-axis magnetic field sensor - specifically, the Go Direct® 3-Axis Magnetic Field Sensor produced by Vernier. Operating based on the Hall effect, this sensor enables precise, spatially resolved measurement of magnetic induction B, allowing students to visualize magnetic field variations in 3D clearly [5,6]. Moreover, the sensor is portable, compatible with mobile devices, and works with free software, providing a modern and engaging laboratory experience for students [7,8]. Thus, by employing components that are widely accessible (as the magnetic field sensor) and hand-made manufactured as the solenoid, the experiment remains accessible and replicable in diverse educational environments. The procedure involves automatically performing repeated measurements of B for each set current value, determining both the mean value of and the uncertainty of this mean. This approach allows indirect determination of the solenoid’s inductance, leveraging the fundamental relationship derived from Faraday’s law. Moreover, students experimentally test the ideal solenoid approximation by verifying the uniformity and homogeneity of the magnetic field within the solenoid key assumptions that simplify theoretical calculations. After obtaining the experimental data, the next critical educational step is to analyze the results accurately. To achieve this, regression analysis methods - Weighted Least Squares (WLS) is employed. The statistical technique help quantify the relationship between electric current and magnetic induction and provide a structured approach for handling experimental uncertainties [9-14].

The paper is structured as follows: First, the theoretical framework, including Faraday’s law and the concept of inductance, is briefly reviewed. Next, the experimental apparatus and procedure are thoroughly described, providing detailed instructions suitable for educational laboratories. Subsequently, the regression methodology is clearly presented, detailed experimental results and data analysis are then discussed, including step-by-step pedagogical instructions for implementing these regression methods. Finally, the conclusion summarizes the study's pedagogical value, the practical efficacy of the experimental method, and the scientific insights gained. Overall, this integrated approach deepens students' understanding of electromagnetic induction, inductance measurement, and data analysis in experimental physics.

2. THEORETICAL APPROACH

To derive the precise theoretical relationship needed to support our experimental setup, we will utilize Faraday's law of electromagnetic induction, which explicitly relates the induced electromotive force (EMF) to the rate of change of magnetic flux, together with the concept of self-induction, describing how an induced EMF arises from the rate of change of electric current flowing within the circuit [1-3]. The primary aim of this derivation is to clearly establish the inductance of the solenoid as a direct and measurable relationship linking variations in the magnetic induction B to corresponding changes in the magnitude of the electric current I, thereby facilitating straightforward and accurate experimental validation.

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t} = -N\frac{S\Delta B}{\Delta t} = -L\frac{\Delta I}{\Delta t} \Rightarrow L = NS\frac{\Delta B}{\Delta I} \qquad (1)$$

Here ΔB and ΔI are the corresponding finite differences, $\Phi = BS$ is the magnetic flux, S is the solenoid's cross-sectional area. From this formula, it follows that if we know the dependence of magnetic induction on current at least in two points, we can determine the inductance L of the solenoid.

In the context of a solenoid, inductance quantitatively characterizes how effectively the solenoid's magnetic field depends on and responds to changes in the electric current flowing through it. At the same time, inductance establishes a direct connection between the geometric and structural parameters of the solenoid and its electromagnetic behavior.

To illustrate this relationship theoretically, we will first consider the case of an ideal solenoid, one that is sufficiently long, tightly wound, and uniformly constructed. Under these idealized conditions, the magnetic field inside the solenoid's central region can be considered homogeneous. For educational completeness and deeper theoretical understanding, the magnetic induction in an ideal solenoid can be derived using Ampère's circuital law, illustrating clearly the relationship between magnetic field and electrical current in closed loop.

As an alternative method for determining inductance, we now aim to derive a formula that expresses the inductance L exclusively in terms of the solenoid's geometric parameters, without involving direct measurements of magnetic induction B or current I. To achieve this, we will first express the inductance in the form $L = NSB/I$, where we will substitute B using the formula for the magnetic field in an ideal solenoid. Thus, we obtain:

$$B = \mu_0 \frac{N}{l} I \Rightarrow L = \mu_0 \frac{N^2}{l} S \tag{2}$$

where $\mu_0 = 4\pi \times 10^{-7}$ H/m, N is the number of turns, l is the length, and I is the current through the solenoid What is the difference between the two formulas (1) and (2) from a pedagogical point of view? The first formula presents inductance as a quantity related to the dynamic relationship between the magnetic field and the current. The second formula relates inductance only to the stationary parameters depending on the construction of the solenoid.

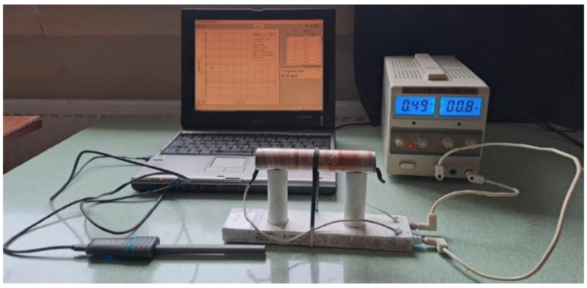
For students studying electromagnetism, these two relationships are valuable because, on one hand, they allow analysis of the dynamic relationship between the magnetic field (magnetic induction) and the current, and on the other hand, they connect this relationship directly to the purely physical characteristics of the solenoid.

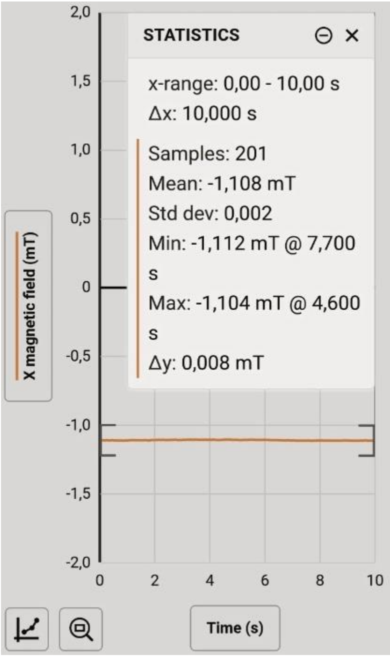
Comparing these two approaches provides valuable insights into the relationship between dynamic and structural parameters, helping students to appreciate the practical implications of theoretical models.

To accurately interpret the experimental measurements of magnetic induction B as a function of current I, regression analysis methods - specifically, Weighted Least Squares (WLS), is utilized.

3. EXPERIMENT

The selection of accessible, affordable equipment and a straightforward experimental methodology ensures that this setup can be easily replicated in a standard university laboratory, making it highly suitable for educational purposes. The components of the setup are as follows: Solenoid with length $l = (0.122 \pm 0.001)[m]$, number of turns $N = (214 \pm 1)$ and cross-sectional area $S = 0.0005\left[m^2\right] \pm 1.59\%$; Current Source: An adjustable laboratory DC power supply is employed, capable of delivering current in the range of 0 to 2[A]. The power supply features a built-in ammeter with an instrumental error of 10^{-2} [A], ensuring sufficient precision for educational applications. Magnetic Field Sensor: Magnetic induction is measured using the Go Direct® 3-Axis Magnetic Field Sensor by Vernier, which operates based on the Hall effect. This sensor enables direct measurement of the magnetic field inside the solenoid with a typical standard measurement error on the order of 10^{-6} [T] calculated automatically by the accompanying software. Data Acquisition Device: The sensor connects to either a computer or an Android mobile device for data acquisition. The corresponding software provided by Vernier is available in both Windows and Android versions, facilitating flexible usage. Figure 1 illustrates the experimental setup in its configuration with a laptop. The magnetic field sensor, resembling a pen-like probe, is positioned beneath the solenoid. Its sensitive Hall sensor is located at the tip, and a directional arrow indicates the positive detection axis of the magnetic induction vector. The sensor communicates with the laptop via a USB connection, utilizing Vernier's dedicated software for data collection and visualization.





STATISTICS

x-range: 0,00 - 10,00 s
Δx: 10,000 s
Samples: 201
Mean: -1,108 mT
Std dev: 0,002
Min: -1,112 mT @ 7,700 s
Max: -1,104 mT @ 4,600 s
Δy: 0,008 mT

Fig. 1 (left side). The experimental setup in a configuration with a USB connection between the sensor and the computer.

Fig. 2. (right side) A view from the software (Android version) that graphically displays one set of measurements from the sensor.

In addition, the sensor offers Bluetooth connectivity, enabling wireless operation with Android devices. This feature provides a valuable perspective for modern laboratory practice, as it allows students to use an Android smartphone or tablet, together with the free Vernier application, as a portable and intuitive visualization and data acquisition tool [6]. The flexibility to collect, display, and analyze data directly on a mobile device enhances accessibility and convenience, making the experimental setup even more adaptable for diverse educational environments [7,8]. Figure 2 presents a screenshot from the phone Android application, demonstrating its functionalities. The software allows the user to define measurement intervals and sample rates, automatically record data, and display graphical representations of the measured magnetic induction over time.

Moreover, the software performs real-time statistical analysis, providing mean values and calculating the standard deviation for each series of measurements. For every data point used in the experiment, the software aggregates a set of automatic readings (e.g., multiple samples over a 10-second interval, as shown in Figure 2) corresponding to a constant current value. This automated process significantly reduces user-induced errors and ensures reproducibility.

The measured magnetic induction values are presented with their correct sign conventions. For instance, the negative values shown in Figure 2 indicate that the magnetic induction vector is oriented in the direction opposite to the positive axis defined by the sensor's casing. This directional sensitivity is an essential aspect of the experiment, providing students with immediate visual confirmation of the magnetic field's orientation relative to the solenoid and the sensor's detection axis.

3.1. METHODOLOGY

The experimental procedure is designed to provide a clear and systematic approach to measuring the inductance of a solenoid using the principles of electromagnetic induction. The methodology consists of the following main steps:

1. Current Variation and Control: The solenoid is powered by an adjustable, stabilized DC power supply, which allows precise control of the electric current. For the purposes of this experiment, the current is incrementally varied from 0.1[A] to 1.2[A] in uniform steps of 0.1[A].

2. Verification of Magnetic Field Homogeneity: Prior to recording measurement data, it is essential to verify that the magnetic field within the region of interest inside the solenoid is homogeneous. This is accomplished by moving the magnetic field sensor (magnetometer) along the longitudinal axis and transversely across the cross-section of the solenoid.

3. Measurement of Magnetic Induction: For each discrete value of current, the magnetic induction B inside the solenoid is measured using the Go Direct® 3-Axis Magnetic Field Sensor. The measurement is automated: over a period of 10 seconds, the device performs a set of multiple measurements, from which it calculates the mean value of B and the standard deviation for each current set.

4. Data Visualization and Preliminary Analysis: A graph of the measured magnetic induction B as a function of electric current I is plotted using the collected data points. The relationship $\Delta B/\Delta I$ could be roughly determined (by hand), which corresponds to the slope of the linear dependence.

5. Theoretical Regression Analysis of Experimental Data: To obtain accurate and reliable estimates of the relationship $\Delta B/\Delta I$, the experimental data are analysed using regression analysis methods: Weighted Least Squares (WLS) is employed to

account for the varying measurement uncertainties (standard deviations) associated with each mean value of *B*.

The regression coefficients obtained (slope b=ΔB/ΔI and intercept a) provide a robust estimation of the relationship between *B* and *I*, while also allowing for the quantification of associated uncertainties. The slope b is directly used for the experimental determination of inductance, linking back to Faraday's law (1).

6. Calculation of Inductance: The inductance *L* of the solenoid is then calculated using two complementary methods:

- Experimental determination via regression slope: Based on the regression results from step 5, inductance is calculated using formulas derived from Faraday's law and the physical relationship between *B*, *I*, and the solenoid's geometry.
- Estimation based on solenoid parameters: An alternative calculation of inductance is performed using the solenoid's known structural characteristics (number of turns *N*, length *l*, cross-sectional area *S*) via the standard formula (3) for an ideal solenoid.

7. Validation through Independent Measurement: To validate the experimental results, the inductance of the solenoid is independently measured using a commercial LC-meter. The comparison between the experimentally determined inductance and the direct measurement from the LC-meter serves as an additional verification of the accuracy and validity of the experiment.

This comprehensive methodology integrates theoretical understanding, experimental practice, and statistical data analysis, providing students with valuable experience in applying fundamental electromagnetic concepts and interpreting real-world measurement data.

3.2. EXPERIMENTAL DATA AND RESULTS

After collecting and processing the experimental data, the next step is to analyze the relationship between magnetic induction *B* and electric current *I* in order to extract the inductance *L* of the solenoid. The resulting graph, presented in Figure 3, reveals a clear linear dependence, as theoretically expected based on the linear dependence between current and magnetic induction in an ideal solenoid.

In Figure 3, each black point represents the mean value of the magnetic induction, calculated from a series of repeated measurements corresponding to a specific current value (a total of 13 data points). Across the entire range of current variation, up to 1.2[A], the experimental data have been fitted using regression analysis. On the right-hand scale of the figure, the residual values are also displayed. These residuals generally represent the differences between the predicted magnetic induction values from the regression fit and the experimentally obtained data. As can be observed, the fit accurately represents the experimental results within the expected level of precision.

In this experiment, the dependent variable - magnetic induction *B* was measured as a function of an independent variable – electrical current *I*. Both variables were subject to instrumental uncertainties (10^{-3}mT and 10^{-2}A) and the magnetic induction was additionally affected by random (statistical) uncertainty inherent in the measurement process. Although the instrumental uncertainty of the independent variable *I* was non-zero, it was considered small relative to the range of variation of *I*. To obtain the overall uncertainty for each mean value of *B*, the instrumental and statistical uncertainties were combined, treating them as independent contributions. This approach ensures a comprehensive assessment of measurement reliability for subsequent data analysis [9-12,14].

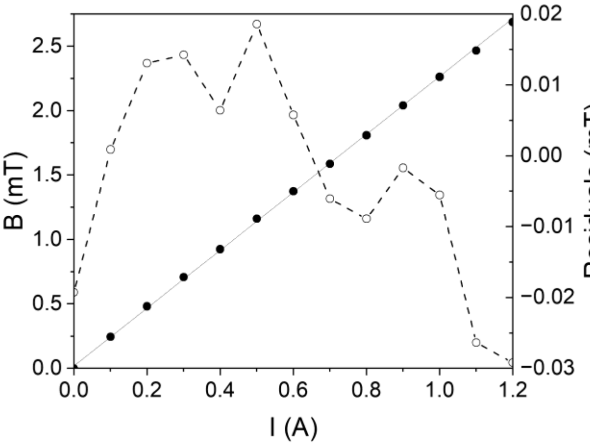


Fig. 3. Dependence of magnetic induction on current: the experimental data shown as black circles are fitted using a linear function. The residuals (right axis) are presented with dotted line with stars.

The Weighted Least Squares (WLS) method accounts for the varying uncertainties of each mean value of *B* by assigning weights inversely proportional to their variances. This is particularly important in our experiment, where each *B* mean is calculated from multiple repeated measurements, resulting in different standard deviations. By emphasizing more precise data and reducing the influence of less reliable points, WLS produces more accurate estimates of the slope and intercept, especially when measurement errors differ across data points. In this analysis, the regression parameters are determined by minimizing the weighted sum of squared residuals (residuals are defined as the differences between the mean values obtained from repeated measurements and the corresponding values predicted by the regression model), where each residual is weighted by the inverse variance of its mean value.: $\chi^2 = \sum_{i=1}^{n}\left(\bar{B}_i - Bp_i\right)^2 / \sigma_i^2$ where $Bp_i = a + bI_i$. Here the calculated regression parameters are the intercept – *a*, and slope - *b* of the desired linear dependency. These residuals provide insights into the adequacy of the linear model and possible systematic deviations or errors. Small residuals (as obtained here) confirm a good fit. Practical formulas that can be directly used for these calculations of *a* and *b* and their uncertainties σ_a and σ_b are [9-12,14]:

$$a=\left(\sum_{i=1}^{n} I_i^2 w_i \sum_{i=1}^{n} \bar{B}_i w_i - \sum_{i=1}^{n} I_i w_i \sum_{i=1}^{n} I_i \bar{B}_i w_i\right)\Big/\Delta\,;\ \sigma_a=\sqrt{\sum_{i=1}^{n} w_i I_i^2 \Big/\Delta}$$

$$b=\left(\sum_{i=1}^{n} w_i \sum_{i=1}^{n} I_i \bar{B}_i w_i - \sum_{i=1}^{n} I_i w_i \sum_{i=1}^{n} \bar{B}_i w_i\right)\Big/\Delta\,;\ \sigma_b=\sqrt{\sum_{i=1}^{n} w_i \Big/\Delta} \qquad (3)$$

$$\Delta=\sum_{i=1}^{n} w_i \sum_{i=1}^{n} I_i^2 w_i - \left(\sum_{i=1}^{n} I_i w_i\right)^2$$

To simplify the calculation of the regression coefficients *a* (intercept) and *b* (slope), an extended table with columns for all necessary intermediate values should be prepared as column in a table. The necessary intermediate values are: $w_i = 1/\left(\sigma_i^2\right)$ - weights based on measurement uncertainties, $I_i w_i$ - product of the electrical current and weights, $\bar{B}_i w_i$ - product of the magnetic inductions and weights, $I_i^2 w_i$ - product of the squared independent variable and weights and $I_i \bar{B}_i w_i$ - product of electric currents and magnetic inductions with weights. These values can be directly used for manual calculation or verification of the regression parameters *a* and *b*.

Following above presented weighted linear regression analysis, the coefficients a (intercept) and b (slope) were determined, yielding the following values: $a = (0.019284 \pm 0.000731)mT$ and $b = (2.248255 \pm 0.001381)mT/A$, where the parameter b (the slope) is the quantity ΔB/ΔI that we had to look for. Based on the regression results (eq. 1) and the physical dimensions (eq. 2) of the solenoid, the inductance was calculated: $L(eq.1) = \left(0.241 \pm 0.005\right)mH$ and $L(eq.2) = \left(0.236 \pm 0.006\right)mH$, demonstrating excellent agreement with both methods. Despite the good agreement, we also employed another method for independent verification of the experimental results - direct measurement of the inductance using an LC meter. The returned a result was $\left(0.23 \pm 0.01\right)mH$, further confirming the validity of the result obtained from the laboratory experiment.

4. CONCLUSION

This experiment demonstrates how the relationship between magnetic induction and electric current in an ideal solenoid can be used for the experimental determination of inductance. Unlike typical LC-circuit setups, this method uses DC currents without moving conductors or magnets. The obtained results show excellent agreement with both theoretical predictions and independent measurements with specialized instrumentation. By guiding students through the process of collecting experimental data and performing regression analysis, the exercise allows direct application of fundamental electromagnetic theory to practical, quantitative problems. The experiment not only reinforces students' conceptual understanding of magnetic fields, electromagnetic induction, and self-induction, but also develops essential laboratory skills, including careful measurement, uncertainty analysis, and statistical data treatment. Its accessible methodology, combined with modern tools for automated data collection and analysis, makes it particularly valuable as a laboratory exercise within a general physics course. As such, it provides a concrete, hands-on framework for bridging theory and practice, supporting deeper learning and scientific reasoning that are crucial for future work in both academic and applied physics contexts.

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