

SELF-ORGANIZATION OF THE IONIC RYDBERG STATES FORMATION IN THE INTERACTION WITH SOLID SURFACE

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We study the electron capture from the solid conduction band into Rydberg states of multiply charged ions escaping solid surfaces at intermediate velocities $v \approx 1$ a.u. A quantum two-state vector model enables us to consider the population process as a synergetic problem, by using the real and imaginary part of the mixed flux I as two independent modes. We found that an initial chaos in the time evolution of the phase point $(\text{Re}I, \text{Im}I)$ results in a self-organization around the stationary point, corresponding to the critical ion-surface distance R_c .

Key words: Rydberg states, neutralization, surfaces, self-organization.

1. INTRODUCTION

The multiply charged ions transferred through a thin foil at intermediate velocity ($v \approx 1$ a.u.) capture a conduction band electrons into Rydberg states with principal quantum number $n_A \geq Z$, where $Z \gg 1$ is the core charge of the formed ion. The Rydberg states are populated in the outgoing part of the ionic trajectory and at sufficiently large distances ($R \approx R_c^N$) from the back side of the foil. For these reasons, the time evolution of the active electron quantum state is a rather long process, realized under prominently nonstationary conditions. Basically, the dynamics of this process can be described by several quantum models [1–10], as well as by the classical over-barrier (COB) model [11, 12], or its extended dynamic version [13, 14].

We provide the results indicating that the nonstationary stages of the electron state evolution in the ion-surface system possesses some properties typical for non-linear phenomena. This is possible within the framework of the time symmetrized quantum model formulated by Aharonov *et al.* [15, 16] and as a two-state vector model (TVM) [17–19] in the context of the ion-surface interaction problem. That is, one can consider the non-resonant electron capture process as a synergetic problem [20–22] describing the active electron simultaneously by the state $|\Psi_1(t)\rangle$ evolving from a given initial state (electron in solid) and with an additional state $|\Psi_2(t)\rangle$, which evolves “teleologically” towards a fixed final state (electron in ion). The interplay of the states $|\Psi_1(t)\rangle$ and $|\Psi_2(t)\rangle$ at intermediate time t is taken into account

via a two-state probability amplitude $A(t)$ of finding the electron in the ionic region. The mixed flux $I(t) = dA/dt$ represents a complex two-current through the moving Firsov plane S_F , placed between the solid surface and the outgoing ionic projectile.

We analyze the causal time dependence of the mixed flux $I(t)$ by means of a trajectory in the phase plane $q_1(t) = \text{Re}I(t)$ and $q_2(t) = \text{Im}I(t)$; namely the modes q_1 and q_2 represent relevant quantities for a description of self-organization of the Rydberg states formation in the interaction with solid surface. It appears that behavior of the phase trajectory in the initial stages of the ion-surface interaction is very irregular, which is characteristic of deterministic chaos [23]. However, starting from the critical time onward, a very regular motion of the phase point in the considered complex plane depicts the subsequent time evolution of the mixed flux. The most important correlation between the TVM and the synergetic considerations is in the fact that the neutralization distance R_c^N , which indicate the most probable ion-surface distance for the population process within the framework of the TVM, appears as a parameter $R = R_c$ of the stationary point (q_{1s}, q_{2s}) of the non-linear dynamics in the complex mixed flux plane. This result can be interpreted as a self-organization of the ionic Rydberg states formation during the ion-surface interaction.

2. FORMULATION OF THE PROBLEM

Our attention is focused on the multiply charged ions escaping a conduction solid surface along the z axis perpendicular to the surface, with intermediate velocities $v = Rt \approx 1$ a.u., where R is the instant ion-surface distance. We consider the electron capture into the Rydberg states $|\nu_A\rangle = |n_A, l_A, m_A\rangle$, with low values of the angular momentum quantum number ($l_A \leq 2$).

In accordance with the very concept of the quantum TVM, the state $|\Psi_1(t)\rangle = \hat{U}_1(t_{in}, t)|\Psi_1(t_{in})\rangle$ of the active electron (representative electron) evolves from the initial parabolic state $|\Psi_1(t_{in})\rangle = |\mu_M\rangle = |\gamma_M, n_{1M}, m_M\rangle$, while the state $|\Psi_2(t)\rangle = \hat{U}_2(t_{fin}, t)|\Psi_2(t_{fin})\rangle$ evolves toward the Rydberg state $|\Psi_2(t_{fin})\rangle = |\nu_A\rangle$ at the final time $t = t_{fin}$. By γ_M we denoted the continuous energy parameter of the electron in solid (with energy $E_M = -\gamma_M^2/2$), and n_{1M} and m_M are the corresponding first parabolic quantum number and magnetic quantum number, respectively. The evolution operators $\hat{U}_1(t_{in}, t)$ and $\hat{U}_2(t_{fin}, t)$ are determined by the Hamiltonians $\hat{H}_1(t)$ and $\hat{H}_2(t)$, respectively [24]. The probability of the electron capture into the field of the moving ion, providing that finally it occupies the given Rydberg state, is determined by the two-state probability amplitude $A(t) = \langle \Psi_2(t) | \hat{P}_A | \Psi_1(t) \rangle$. The projecting operator \hat{P}_A in a coordinate representation is given by the Heaviside function $\Theta(z - R + a(t))$, whereas z is the instant electron position and $a = a(t)$ is the position of the Firsov plane S_F in respect to the ion.

The two-state probability amplitude $A(t)$ can be expressed as $A(t) = \int_{t_g}^t I(t) dt$, valid for $t > t_g$, where the mixed flux $I(t)$ is given by the surface integral (Eq. (2.5) in Ref. [17])

$$I(t) = \frac{i}{2} \int_{S_F} \left[\frac{\nabla \Psi_1}{\Psi_1} - \frac{\nabla \Psi_2^*}{\Psi_2^*} - 2iv \left(1 - \frac{da}{dR} \right) \vec{e}_z \right] \Psi_2^* \Psi_1 d\vec{S}, \quad (1)$$

whereas $d\vec{S} = dS \vec{e}_z$, and \vec{e}_z is the unit vector of the z axis. In a definition of the quantity $A(t)$ we took into account that for $t \leq t_g$ the reionization process will completely destroy the formed Rydberg states, so that the population practically begins at $t = t_g$. In the considered low- l Rydberg states [18, 19] it has been possible to extrapolate the expression (1) to $t_g = 0$. However, the phase trajectory in the initial stages of the ion-surface interaction is very irregular, which is a characteristic of deterministic chaos [23]. The electron exchange process is stabilized at larger ion-surface distances, where it is characterized by the intermediate transition probability per unit γ_M as $T_{\mu_M, \nu_A}(t) = |A(t)|^2$. By integration over all possible γ_M -values, together with summation over n_{1M} and m_M , we get the intermediate population probability $P_{\nu_A}(t)$. The total rate [18] for the population process can be defined by the following expression:

$$\tilde{\Gamma}_{\mu_M, \nu_A} = \frac{1}{T_{\mu_M, \nu_A}(t_{fin})} \frac{dT_{\mu_M, \nu_A}(t)}{dR}. \quad (2)$$

We note that the position of the Firsov plane S_F in the expression (2) for the mixed flux is determined by the variation requirement $\delta P_{\nu_A}(t)/\delta a = 0$ together with boundary conditions $\delta a(t_{in}) = 0$ and $\delta a(t_{fin}) = 0$ [18, 25].

As a complex quantity, the mixed flux $I(t)$ gives us a possibility to consider its real part $q_1(t)$ and imaginary part $q_2(t)$ as two generalized coordinates (modes) of the classical self-organizing dynamics. The corresponding phase trajectory, parametrically defined by $R = vt$, will be considered in the asymptotic region $R \gg 1$ a.u., characteristic for the formation of the Rydberg state (n_A, l_A, m_A) .

Restricting to the asymptotic region, we can express the time derivative $\dot{I}(t)$ of the mixed flux by using the simplest non-linear model:

$$\dot{I}(t) = a_1 I(t) + a_2 I^2(t). \quad (3)$$

Using the notation $q_1(t) = \text{Re}I(t)$ and $q_2(t) = \text{Im}I(t)$, Eq. (3) overcomes into the following system of equations:

$$\dot{q}_1 = L_{11}q_1 + L_{12}q_2 + N_1(q_1, q_2), \quad (4)$$

$$\dot{q}_2 = L_{21}q_1 + L_{22}q_2 + N_2(q_1, q_2), \quad (5)$$

where $L_{11} = L_{22} = \text{Re}a_1$ and $L_{21} = -L_{12} = \text{Im}a_1$. The non-linear terms in Eqs. (4) and (5) are given by $N_1(q_1, q_2) = (q_1^2 - q_2^2)\text{Re}a_2 - 2q_1q_2\text{Im}a_2$ and $N_2(q_1, q_2) = (q_1^2 - q_2^2)\text{Im}a_2 + 2q_1q_2\text{Re}a_2$. It is obvious that modes $q_1(t)$ and $q_2(t)$ satisfy the system

of first order coupled non-linear differential equations, so we are able to construct a synergetic description of the charge exchange process. The initial condition $q_1(t_{in}) \equiv q_{10} = 0$, $q_2(t_{in}) \equiv q_{20} = 0$, where $t_{in} = 0$ is presumed. The order of magnitude of the coefficients L_{ij} in the asymptotic region determines the self-organization of modes q_1 and q_2 . We have $L_{22} < 0$ and $|L_{21}| \ll |L_{22}|$, so that we can introduce the universal infinitesimal $\varepsilon = L_{21}/L_{22} = -L_{12}/L_{11}$, satisfying the condition $|\varepsilon| \ll 1$. To investigate the self-organization of the considered two modes, it is convenient to distinguish between the order mode and the “stable” (or slaved) mode, which follows immediately the order mode [22]. However, in the considered problem we have $L_{11} = L_{22}$, so that the both modes can be stable (or order). For convenience, we shall treat the $q_2(t) = \text{Im}I(t)$ as a stable mode and $q_1(t) = \text{Re}I(t)$ as an order mode. The system “ q_2 ”, Eq. (5), is damped in the absence of the system “ q_1 ”, Eq. (4), because $L_{22} < 0$.

We can use the adiabatic elimination technique [22] to obtain an explicit and unique solution of the system given by Eqs. (4) and (5). That is, we may solve Eq. (5) for stable mode by putting $\dot{q}_2 \approx 0$, and thus eliminate the stable mode from the system of equations. By this procedure we get the connection $q_2 = \mathcal{F}(q_1)$ between the two modes determined by the following equation: $L_{21}q_1 + L_{22}q_2 + N_2(q_1, q_2) = 0$. With the relation $q_2 = \mathcal{F}(q_1)$, Eq. (4) for the order mode obtains a form of a classical equation of overdamped motion of the form $\dot{q}_1 = F(q_1)$, where F is a generalized force. The stationary solution $q_1 = q_{1s}$ of the former equation, defined by the condition $\dot{q}_1(q_{1s}) = 0$, together with the corresponding value $q_{2s} = \mathcal{F}(q_{1s})$ constitute the stationary point (q_{1s}, q_{2s}) of the considered system. We note that, outside of the adiabatic approximation, the stationary point is defined by the system of equations $\dot{q}_1(q_{1s}) = 0$, $\dot{q}_2(q_{2s}) = 0$. The stationary point represents an equilibrium point of the system. If the nonzero stationary solution $q_{1s} \neq 0$ exists, the system (phase point (q_1, q_2)) from the equilibrium point $(q_{10} = 0, q_{20} = 0)$ at $t = t_{in} = 0$ “has internally decided to produce” a finite quantity $q_{2s} = \mathcal{F}(q_{1s})$, *i.e.*, we have a self-organization in the system. The stationary point (q_{1s}, q_{2s}) characterized a new equilibrium at the time $t > t_{in} = 0$. This point is defined by a system of coupled algebraic equations:

$$L_{11}q_{1s} + L_{12}q_{2s} + N_1(q_{1s}, q_{2s}) = 0, \quad (6)$$

$$L_{21}q_{1s} + L_{22}q_{2s} + N_2(q_{1s}, q_{2s}) = 0. \quad (7)$$

In the considered adiabatic approximation, the stationary point belongs to the phase trajectory $q_2 = \mathcal{F}(q_1)$. The ion-surface distance corresponding to the point (q_{1s}, q_{2s}) can be considered as the critical distance $R = R_c$ for the formation of the particular Rydberg state by the self-organizing procedure, *i.e.*, we have a condition $\dot{q}_2(R_c) = 0$. In this case, the stationary point (q_{1s}, q_{2s}) could be a stable point that corresponds to the formation of a stable ionic Rydberg state, or could be an unstable point that

represents a formed ionic Rydberg state that is not stable due to significant process of reionization; in the second case, the small fluctuations of the system push it away from this unstable point to the new critical point.

3. A SELF-ORGANIZATION OF THE IONIC RYDBERG STATES FORMATION

Further analysis of the self-organization process consists in determination of the stationary point of the system. The first step is to establish the expansion coefficients a_1 and a_2 in expression (3) explicitly. The coefficient a_1 can be obtained from the asymptotic expression of the mixed flux $I(t)$. On the other hand, the coefficient a_2 can be expressed *via* a_1 within the framework of the proper non-linear model for the time derivative of the mixed flux $I(t)$ valid in the asymptotic region $R = R_c$.

We consider the case of low angular momentum Rydberg states, when the expression for the mixed flux is known in the analytic form: Eq. (4.2) in Ref. [17] for the point-like core approximation and Eq. (17) in Ref. [18] for the polarized cores. In the first case we have

$$I(t) \approx I_0 \exp(i\delta) \exp[i\omega t + (\tilde{\gamma}_M - \gamma_{A0})gR - \tilde{\gamma}_M R], \quad (8)$$

where I_0 is a slowly varying real function of time, $\tilde{\gamma}_M = [1 + 3/(4\alpha)][1 + 3/(2\alpha)]^{-1/2} \gamma_M$, $w = (\tilde{\gamma}_M^2 - \gamma_{A0}^2)/2 - v^2(1 - 2g)/2$ and $\gamma_{A0} = Z/n_A$. The parameter $\alpha = \alpha(R) = \gamma_M^2 R / (Z - 1/4)$ is introduced as a scaling parameter to characterize the electron transitions in the very vicinity of the potential barrier top. We note that these electron transitions are responsible for the population of the resonant Rydberg states $n_A = n_{res}$. The parameter $g = a/R = g(\tilde{\gamma}_M, \gamma_{A0}, v)$, which determines the kinematics of the Firsov plane, is explicitly given in Ref. [25] (with $\gamma \rightarrow \tilde{\gamma}_M$). By δ in Eq. (8) we denoted the phase factor that can be considered as a real, time independent quantity. Using Eq. (8) we get an approximate expression for the time derivative of the mixed flux. Comparing the so obtained expression with Eq. (3) we get

$$\text{Re}a_1 = L_{11} = (\tilde{\gamma}_M - \gamma_{A0})gv - \tilde{\gamma}_M v, \quad \text{Im}a_1 = -L_{12} = w. \quad (9)$$

In order to calculate the coefficient a_2 , we propose the exponential model $\dot{\tilde{I}} = 1 - \exp(-k\tilde{I})$, where $\tilde{I} = \lambda I$, $|k\tilde{I}| = |k\lambda I| \ll 1$ and k is a complex constant. In addition, we assume that λ is a real parameter, which is possible only for specific values of the phase factor δ . Using the first two terms in the expansion of the right hand side of the last expression, we obtain the expression of the form (3) provided that $a_2 = -\lambda a_1^2/2$. From the last relation we get $\text{Re}a_2 = -\lambda(L_{11}^2 - L_{12}^2)/2$ and $\text{Im}a_2 = \lambda L_{11}L_{12}$, where λ is at present a free parameter. In the low velocity limit $v \rightarrow 0$, when the electron transitions are isoenergetic, we have $w \rightarrow 0$. In this case we get $L_{11} \rightarrow 0$ and $L_{12} \rightarrow 0$, so that Eqs. (6) and (7) are identically satisfied for any stationary point. For nonresonant electron transitions, characteristic for the

intermediate velocities, we still have that $w \approx 0$. Therefore, we have that $|L_{12}| \approx 0$, which implies the validity of the relation $|\varepsilon| \ll 1$.

A system of equations (6) and (7), which determines the stationary points, can be solved analytically. By elimination of the quantity $q_{1s}^2 - q_{2s}^2$ from the considered system and using the relations for $\text{Re}a_2$ and $\text{Im}a_2$, we get $q_{2s} = q_{1s}\varepsilon/[1 - (1 + \varepsilon^2)\lambda L_{11}q_{1s}]$. Inserting last expression in Eq. (7), we obtain the following equation for scaled stationary point $Q_{1s} = \lambda L_{11}q_{1s}$:

$$Q_{1s} \left\{ 1 + \frac{1}{Q_{1s}(\varepsilon)} - Q_{1s} \left[1 - \frac{\varepsilon^2}{Q_{1s}^2(\varepsilon)} + \frac{1 - \varepsilon^2}{Q_{1s}(\varepsilon)} \right] \right\} = 0, \quad (10)$$

where we used the notation $Q_{1s}(\varepsilon) = 1 - (1 + \varepsilon^2)Q_{1s}$. The solution $q_{1s} = q_{10} = 0$, when $q_{2s} = q_{20} = 0$, corresponds to the initial state of a system with ion-surface distance $R = 0$ as well as to the final state of the system ($R \rightarrow \infty$). The shift of this point from t_{in} to t_c (or from t_{fin} to t_c), where t_c corresponds to the critical ion-surface distances $R_c \gg 1$ a.u., is due to the self-organization in the system.

The nonzero solutions of Eq. (10) are: $Q_{1s}^{(1)} = 2/(1 + \varepsilon^2)$ and $Q_{1s}^{(2/3)} = (1 \pm i\varepsilon)/(1 + \varepsilon^2)$. From these solutions only the first one can be close to the phase trajectory since the last two are physically irrelevant (out of range of Q_{1s}); the second scaled stationary point $Q_{2s} = \lambda L_{11}q_{2s}$ is obviously a form of $Q_{2s} = -2\varepsilon/(1 + \varepsilon^2)$. The parameter λ figuring in the relations $Q_{1s} = \lambda L_{11}q_{1s}$ and $Q_{2s} = \lambda L_{11}q_{2s}$ can be obtained from the explicit expression for the mixed flux, with appropriate choice of the phase factor δ . However, for our further analysis of the critical ion-surface distance R_c , it is sufficient to consider the relation $q_{2s}/q_{1s} = -\varepsilon$, which leads us to the conclusion that the stationary point (q_{1s}, q_{2s}) that belongs to the formation of the stable ionic Rydberg state, is in the intersection of the line $q_2 = -\varepsilon q_1$ and the phase trajectory $q_2 = \mathcal{F}(q_1)$.

4. RESULTS AND DISCUSSION

The exposed method for determination of the critical ion-surface distances R_c will be tested first in the case of ArVIII ion escaping the graphite solid surface in the normal emergence geometry with $v = 1.42$ a.u. We consider the population of the resonant level ($n_A = n_{res}$, $l_A = 1$) by the electron captured from the Fermi level, *i.e.*, we take $\gamma_M = 0.47$ a.u. $\approx \gamma_F$, where $\gamma_F = \sqrt{2\phi}$. We use the values $n_{1M} = m_M = m_A = 0$, which give the main contribution to the population probability. In Fig. 1(a) we present the total rate $\tilde{\Gamma}_{\mu_M, \nu_A}$ *via* ion-surface distance R . We consider the population of the resonant Rydberg states with $n_A = 9, 10$, and 11 of the ArVIII ion interacting with solid surfaces with work functions $\phi = 5$ eV, 4 eV, and 3 eV, respectively. In Fig. 1(b) we present the stable mode $q_2 = \text{Im}I$ *via* R for the same ionic characteristics as in Fig. 1(a). From Fig. 1 we can see that the neutralization

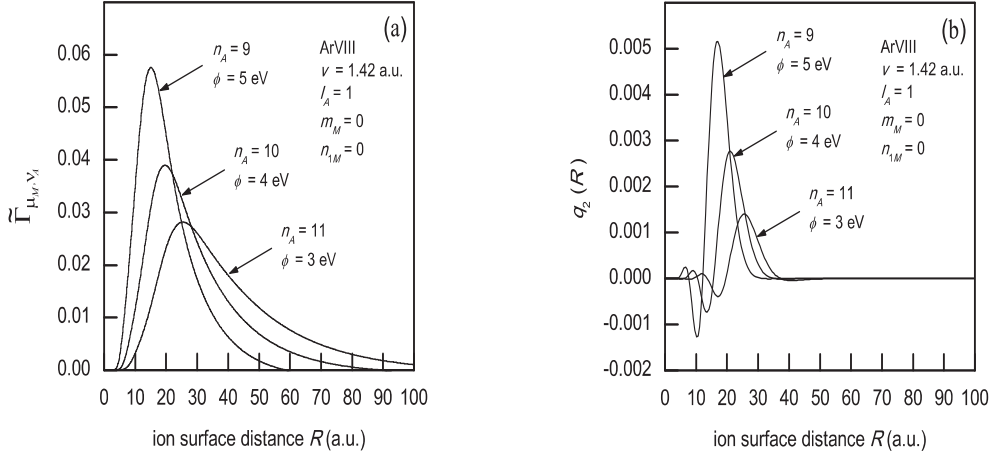


Fig. 1 – Comparison of the neutralization distances R_c^N obtained from (a) the positions of maxima of the total rates $\tilde{\Gamma}_{\mu_M, \nu_A}$ with (b) the critical distances R_c recognized in self-organized procedure as $\dot{q}_2(R_c) \approx 0$ in the case of ArVIII.

distances R_c^N , obtained from the positions of maxima of the total rates $\tilde{\Gamma}_{\mu_M, \nu_A}$, are comparable with the critical distances R_c obtained in self-organized procedure, *i.e.*, imposing the condition $\dot{q}_2 \approx 0$ on the stable mode. We recall that the critical distances R_c for the electron capture estimated by $R_c \approx R_c^N = 2\kappa\alpha_R / [\tilde{\gamma}_A g + \tilde{\gamma}_M(1-g)]$ within the framework of the TVM, determine the ion-surface distances characteristic for the electron transitions in the very vicinity of the potential top. Parameters κ and α_R in the former expression are explicitly given in Ref. [17]; the energy parameter $\tilde{\gamma}_A$ corresponds to the considered Rydberg state in the field of polarized ionic core. The stable mode $q_2 = \text{Im}I$ is defined up to the phase factor δ of the mixed flux. However, the R_c values defined by the condition $\dot{q}_2(R_c) = 0$ are practically independent on the value of δ . By this procedure we implicitly determine the factor δ corresponding to the exponential model with real λ parameter. This circumstance enables us to present the evolution of the phase point and the position of the stationary point.

The full insight in the self-organization in the formation of the Rydberg state can be seen from the time evolution of the phase point in the complex mixed flux-plane. The stable point (q_{1s}, q_{2s}) is defined in the intersection of the line $q_2 = -\varepsilon q_1$ with phase trajectory. If we bear in mind Eq. (9), the quantity ε is given by $\varepsilon = w[(\tilde{\gamma}_M - \gamma_{A0})gv - \tilde{\gamma}_M v]^{-1}$. From the last relation based on the mixed flux, we can see that ε depends on R . For small values of R , corresponding to the initial stages of the process, we are far from stable region. However, in the asymptotic region we have basically $\varepsilon \approx \text{const} \ll 1$.

In Fig. 2(a) we present the initial stages of the phase trajectory, considering the vicinity of the initial stationary point $(0, 0)$. We are able to recognize an irregularity

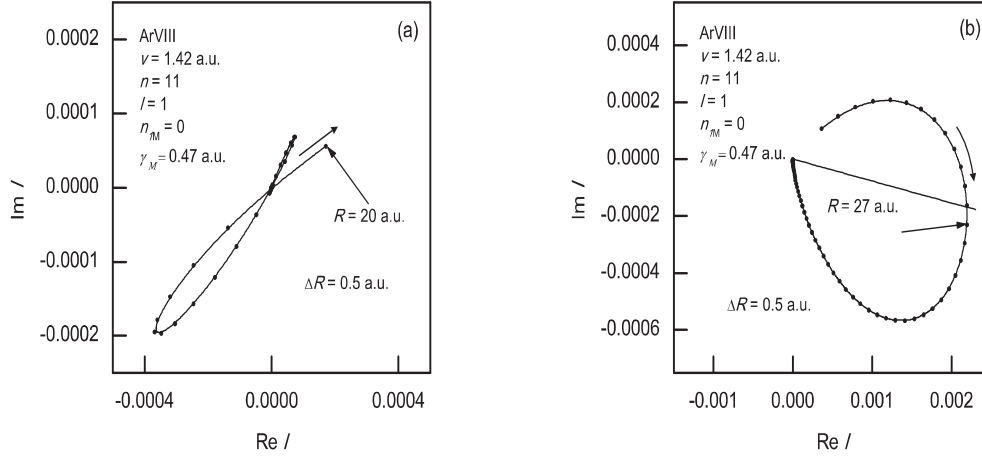


Fig. 2 – (a) The initial and (b) the self-organizing stages of the phase trajectory $q_2 = \text{Im}I$ via $q_1 = \text{Re}I$ in the case of ArVIII.

of the phase trajectory. This circumstance suggests that a formation of the stable bound states of Rydberg ions is not possible in the initial parts of their trajectory. In Fig. 2(b) we show the asymptotic case of the ion-surface distances where the phase points are organized in a smooth part of the phase curve. In the same figure we recognize the stationary point obtained in the intersection of the line $q_2 = -\varepsilon q_1$ with the phase trajectory.

So far our consideration is devoted to $\gamma_M = \gamma_F$. The contribution of other initial states can be also considered within the self-organizing procedure. That is, by taking into account that the electron-capture transition probability is determined by $T_{\mu_M, \nu_A} = |\int_0^t I(t) dt|^2$, we can establish the connection between the probability of the electron capture process and the area enclosed by the phase trajectory. Figure 3(a). shows the ArVIII phase trajectories for $\gamma_M = 0.47, 0.57, 0.67$, and 0.77 a.u., $n_A = 11$, $l_A = 1$, $n_{1M} = 0$, with the velocity $v = 1.42$ a.u. of the ionic projectile. For $\gamma_M = 0.47$ a.u., we have the maxima of the covered area that is in agreement with the fact that the maximal transition probability T_{μ_M, ν_A} is from the Fermi level, which is also in agreement with the available beam-foil experiments [26, 27].

In Fig. 3(b) we present the neutralization distance R_c for the population of the resonant Rydberg states of the ArVIII, KrVIII and XeVIII ions escaping the various surfaces. We consider the normal emergency geometry, taking for the ionic velocity the value $v = 1.42$ a.u. The distances R_c obtained using the SO methodology (full curves) are in a good agreement with the TVM prediction (dots). Finally, the ion-surface distance of formation of Rydberg states above the surface can be estimated by employing the COB model for resonant electron exchange between metal and ion. For $Z = 8$ ions, the obtained first neutralization distance $R_c^{class} = \sqrt{8Z + 2}/(2\phi)$ [28]

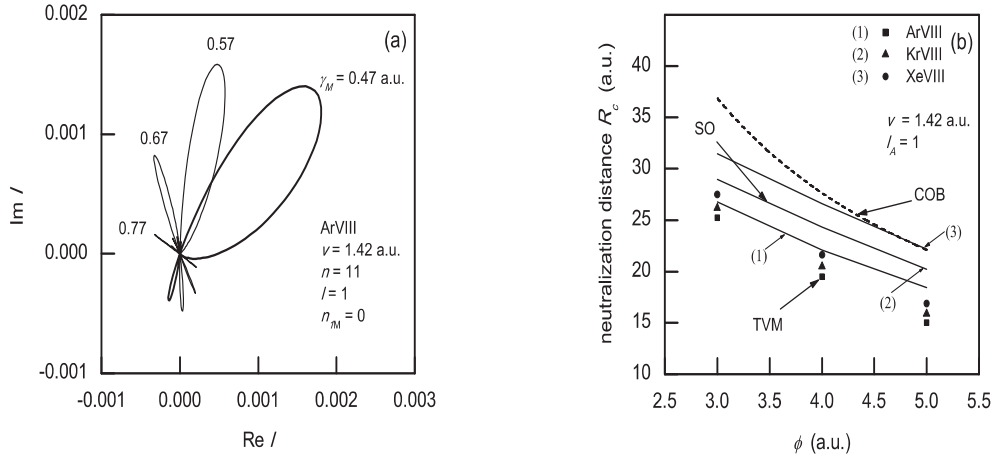


Fig. 3 – (a) The phase trajectory in the case of ArVIII ion escaping the solid surface ($\phi = 3$ eV, $v = 1.42$ a.u.) for $\gamma_M = 0.47, 0.57, 0.67$, and 0.77 a.u.; (b) Neutralization distances R_c for the different ions with the same core charge $Z = 8$ escaping the solid surface with $v = 1.42$ a.u., for electron capture into the Rydberg state: ArVIII $n_A = 11, 10, 9$, KrVIII $n_A = 12, 11, 10$ and XeVIII $n_A = 13, 12, 11$ for $\phi = 3, 4, 5$ eV, respectively ($l = 1, m_A = 0$). The dots correspond to the predictions of the TVM, the dashed curve presents the first neutralization distance proposed by the COB method and full curves present the critical ion-surface distances R_c obtained using the self-organizing methodology.

(dashed curve) is in very well agreement with the SO method, particularly for pure metal.

5. CONCLUDING REMARKS

We have opened up a new way to investigate electron capture from the solid into Rydberg states of highly charged ions. The synergetic approach has established itself as a powerful method of investigation of population process. It is a direct consequence of the applied teleological model with two states; only in such a case, one can define the mixed flux as a complex quantity with both $\text{Re}I(t) \neq 0$ and $\text{Im}I(t) \neq 0$. It has been found that these two quantities play a role of the order q_1 and the stable q_2 modes in the self-organization of the ionic Rydberg states formation in the presence of the metal. The obtained position of the stable point and the corresponding ion surface distance, means that the self-organization occurs when the information coming from the past and the information arriving from the future are in agreement as seen from the standpoint of the mixed flux.

In the low angular momentum case, the mixed flux can be expressed in the analytical form and it has been possible to develop an analytical method of investigation of the self-organization process. For large l values one has to develop a numerical

method. The renormalization of the quantum teleology, as a method developed in order to include the reionization, effect on the formation of the ionic Rydberg states in a specific way. For the observable Rydberg states, the self-organization is survived with the position of the stable point shifted toward larger distance; in the case of short-lived Rydberg states, the self-organization is completely destroyed. In the fast ionization case, the distance R_c is shifted to the very large distances where the probability of the electron capture is negligible. Electron capture into the Rydberg states from the surface covered by the screening thin film in the presence of the weak electric field, may be relevant for further synergetic investigation [29].

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